CSCI4155/CSCI6505: Assignment 1

This is an individual assignment. Assignments must be submitted on paper at the beginning of next class, September 17, 2013. Late assignments not accepted.

1. What is the derivative of the function $f(x; \beta) = x \tanh(\beta x)$? [**1**]

2. What is the inverse of the matrix $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$? [**1**]

3. Using Matlab, plot two figures with a histogram each of random numbers drawn from the following distributions. The first histogram should represent a Chi-square distribution. The second histogram should represent the Trappenberg distribution. The Trappenberg distribution is given by

$$p(x) = \begin{cases} a_n \|\sin(x)\| & \text{for } 0 < x < n\pi/2 \\ 0 & \text{otherwise} \end{cases}$$

for $n = 5$. Also provide the numerical values for the mean, the variance, and skewness for each of these distributions. Note that figures must always contain labels for the axes. [**3**]

4. (From Thrun, Burgard and Fox, Probabilistic Robotics) A robot uses a sensor that can measure ranges from $0m$ to $3m$. For simplicity, assume that the actual ranges are distributed uniformly in this interval. Unfortunately, the sensors can be faulty. When the sensor is faulty it constantly outputs a range below $1m$, regardless of the actual range in the sensor's measurement cone. We know that the prior probability for a sensor to be faulty is $p = 0.01$.
   Suppose the robot queries its sensors N times, and every single time the measurement value is below $1m$. What is the posterior probability of a sensor fault, for $N = 1, 2, ..., 10$. Formulate the corresponding probabilistic model. [**3**]

5. Given are four Bernoulli distributed random variables X_1, X_2, X_3 and Y. The conditional probability of random variables X_i on Y is given by $p(x_i|y) = 0.2$ and $p(x_i|\neg y) = 0.6$, and all x_i are conditionally independent of each other given Y. The marginal probability of Y is $p(y) = 0.3$. What is the probability of Y given X_1 =true and X_2 =true and X_3 =false ? [**2**]