
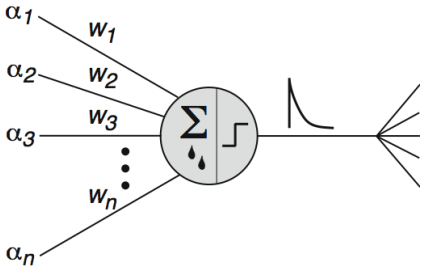


1 The leaky integrate-and-fire neuron 

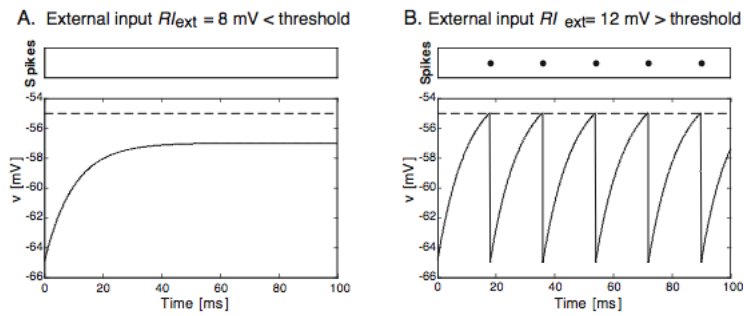
$$\tau_m \frac{dv(t)}{dt} = -(v(t) - E_L) + RI(t), \quad (1)$$

$$v(t^f) = \vartheta. \quad (2)$$

$$\lim_{\delta \rightarrow 0} v(t^f + \delta) = v_{\text{res}}, \quad (3)$$


2

IF simulation



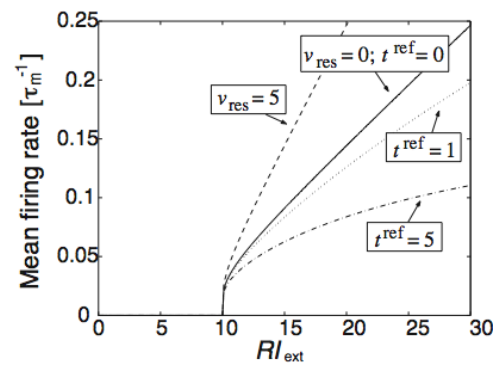
3

IF gain function



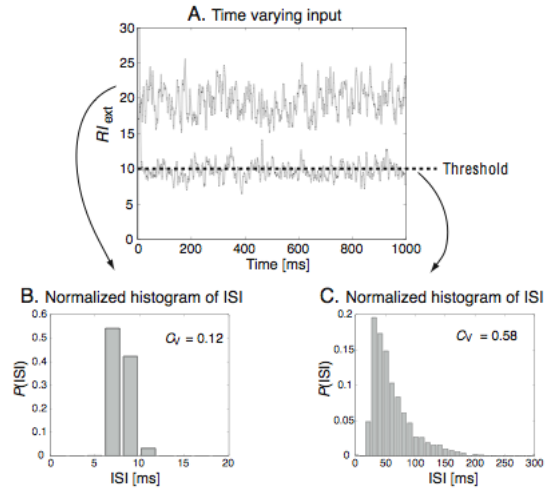
The inverse of the first passage time defines the **firing rate**:

$$\bar{r} = (t^{\text{ref}} - \tau_m \ln \frac{\vartheta - RI}{v_{\text{res}} - RI})^{-1}$$



4

IF resistance to noise



5

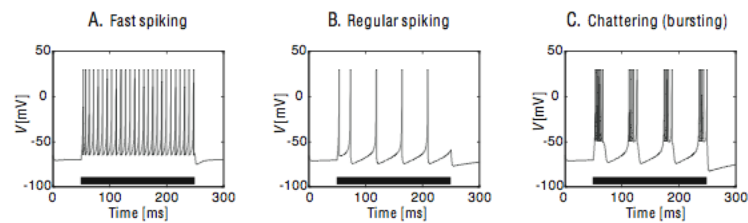
The Izhikevich neuron



$$\frac{dv(t)}{dt} = 0.04v^2(t) + 5v(t) + 140 - u + I(t)$$

$$\frac{du(t)}{dt} = a(bv - u)$$

$$v(v > 30) = c \text{ and } u(v > 30) = u + d$$



6

The McCulloch-Pitts neuron



$$h = \sum_i x_i^{\text{in}}$$

$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$

BULLETIN OF
MATHEMATICAL BIOPHYSICS
VOLUME 5, 1943

A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

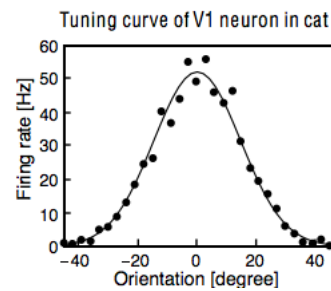
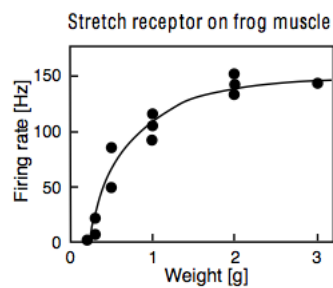
WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INSTITUTE,
AND THE UNIVERSITY OF CHICAGO

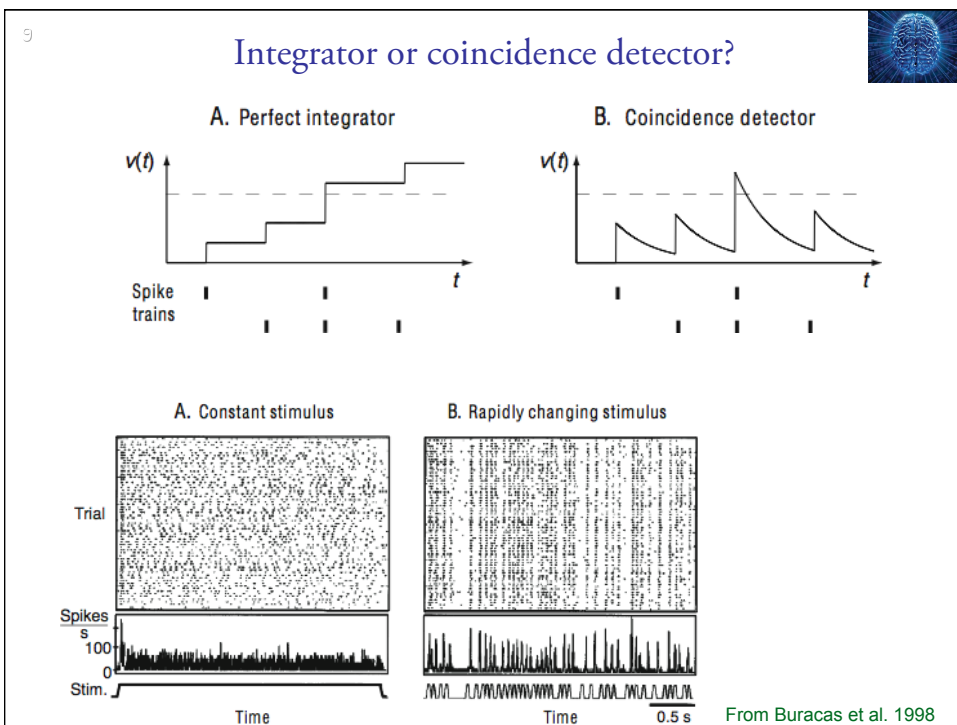
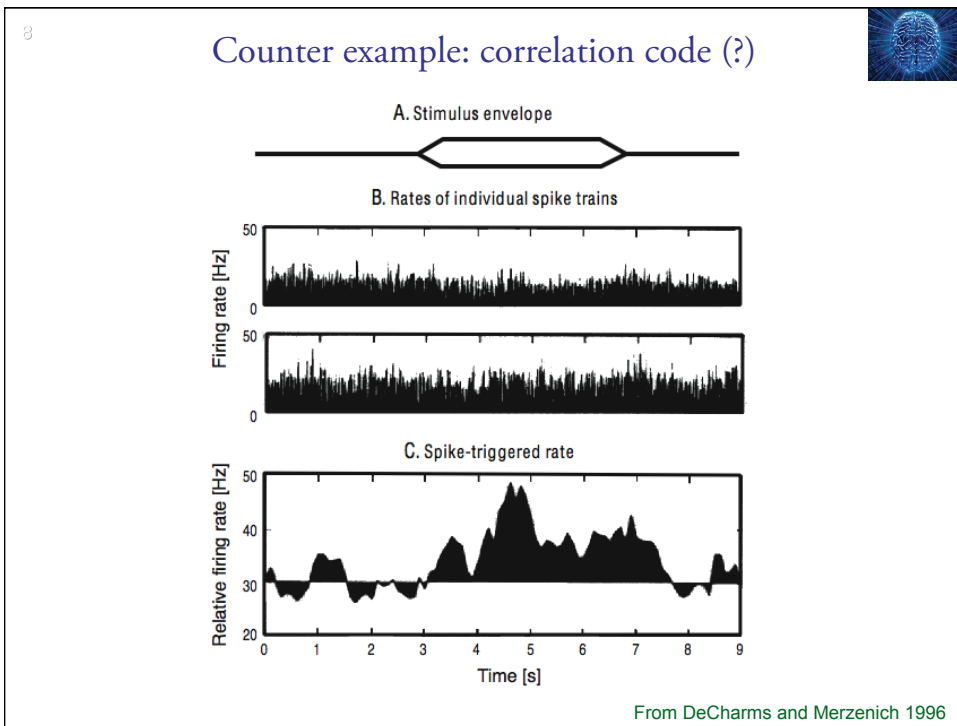
Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying

7

The firing rate hypothesis



Edgar Adrian
The Nobel Prize in Physiology or Medicine 1932



10

Population model

Temporal averaging

Population averaging

11

Population dynamics

For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

$$\tau \frac{dA(t)}{dt} = -A(t) + g(RI^{\text{ext}}(t)).$$

Gain function:

$$g(x) = \frac{1}{t^{\text{ref}} - \tau \log(1 - \frac{1}{\tau x})},$$





A. Activation function for population average in adiabatic limit

B. Activation function of hippocampal pyramidal neuron

12

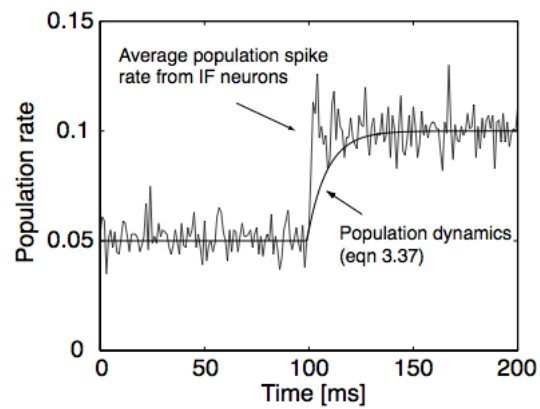
Other gain functions



Type of function	Graphical represent.	Mathematical formula	MATLAB implementation
Linear		$g^{\text{lin}}(x) = x$	<code>x</code>
Step		$g^{\text{step}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	<code>floor(0.5*(1+sign(x)))</code>
Threshold-linear		$g^{\text{theta}}(x) = x \Theta(x)$	<code>x.*floor(0.5*(1+sign(x)))</code>
Sigmoid		$g^{\text{sig}}(x) = \frac{1}{1+\exp(-x)}$	<code>1./(1+exp(-x))</code>
Radial-basis		$g^{\text{gauss}}(x) = \exp(-x^2)$	<code>exp(-x.^2)</code>

13

Fast population response



Further readings



- Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.
- Wulfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.
- Eugene M. Izhikevich (2003), **Simple Model of Spiking Neurons**, in **IEEE Transactions on Neural Networks**, 14: 1569–1072.
- Eugene M. Izhikevich (2004), **Which model to use for cortical spiking neurons?**, in **IEEE Transactions on Neural Networks**, 15: 1063–1070.
- Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in **Bulletin of Mathematical Biophysics** 7:115–133.
- Huge R. Wilson and Jack D. Cowan (1972), **Excitatory and inhibitory interactions in localized populations of model neurons**, in **Biophys. J.** 12:1–24.
- Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.