

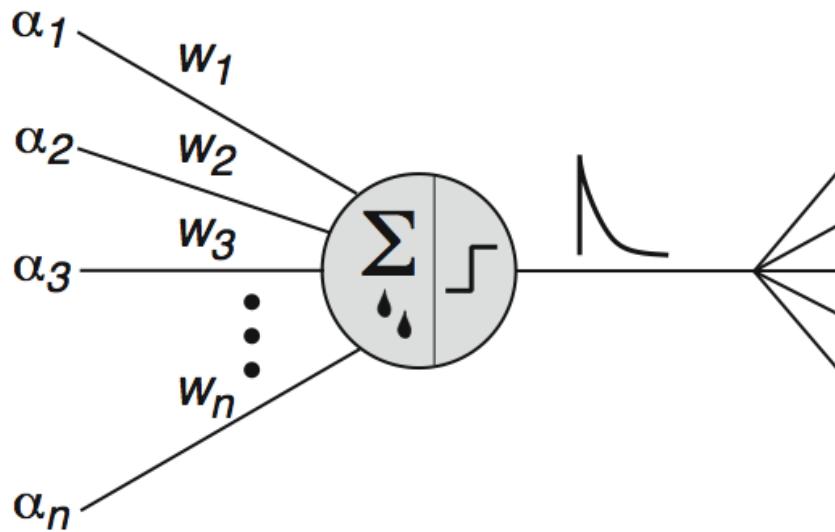
The leaky integrate-and-fire neuron



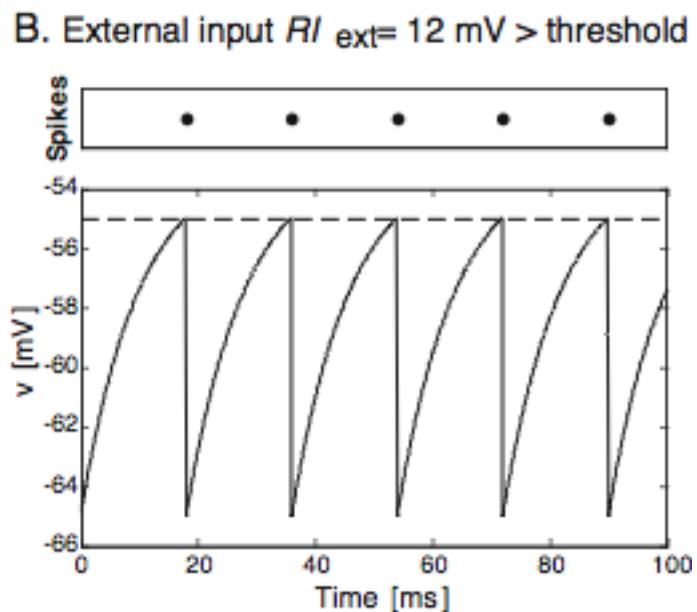
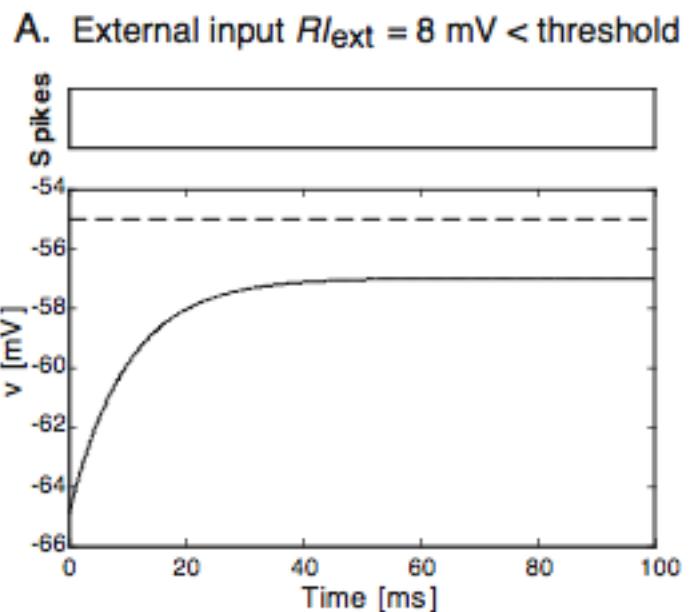
$$\tau_m \frac{dv(t)}{dt} = -(v(t) - E_L) + RI(t), \quad (1)$$

$$v(t^f) = \vartheta. \quad (2)$$

$$\lim_{\delta \rightarrow 0} v(t^f + \delta) = v_{\text{res}}, \quad (3)$$



IF simulation

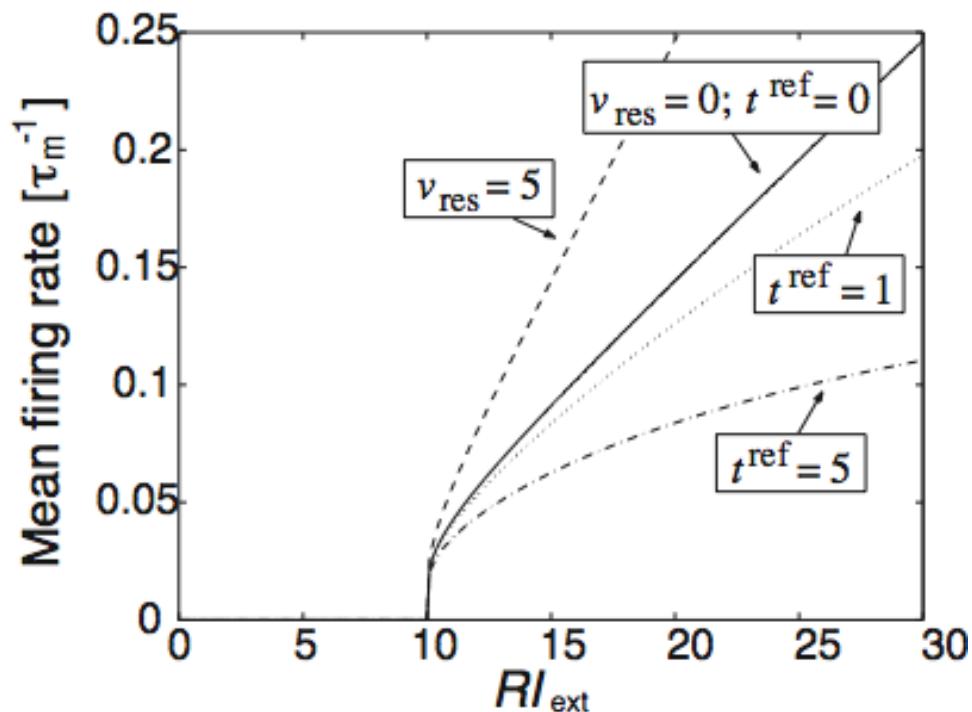


IF gain function

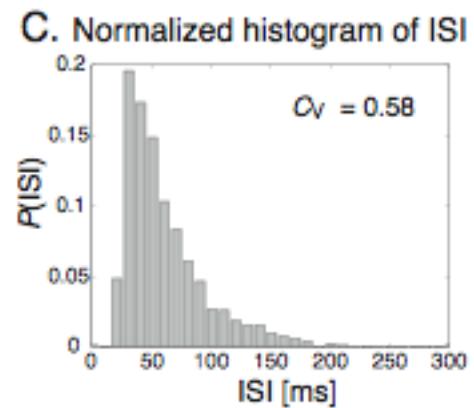
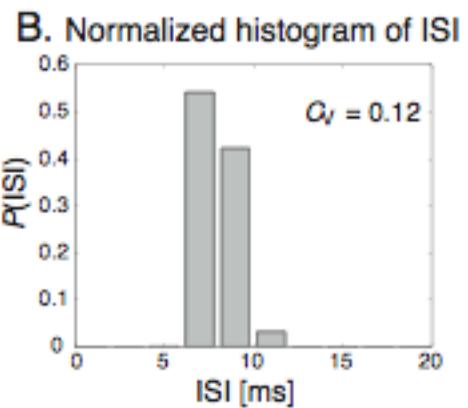
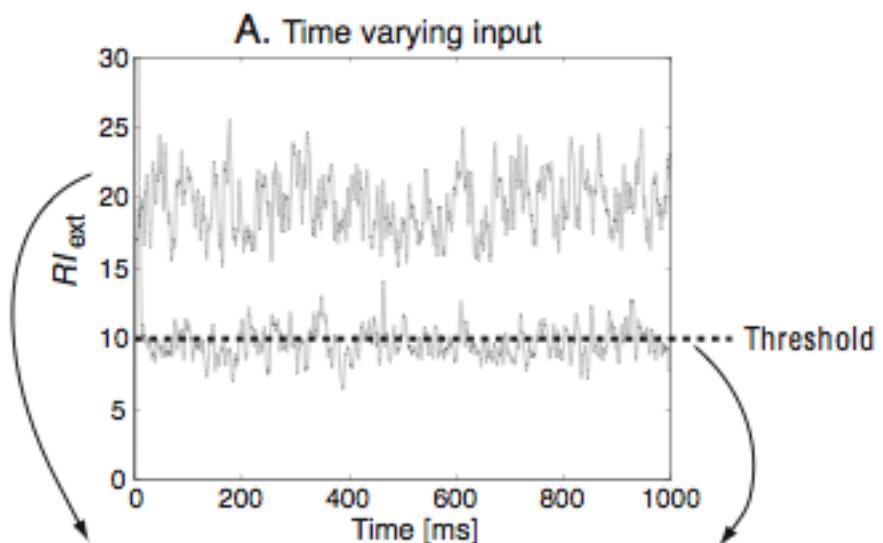


The inverse of the first passage time defines the **firing rate**:

$$\bar{r} = (t^{\text{ref}} - \tau_m \ln \frac{\vartheta - RI}{v_{\text{res}} - RI})^{-1}$$



IF resistance to noise

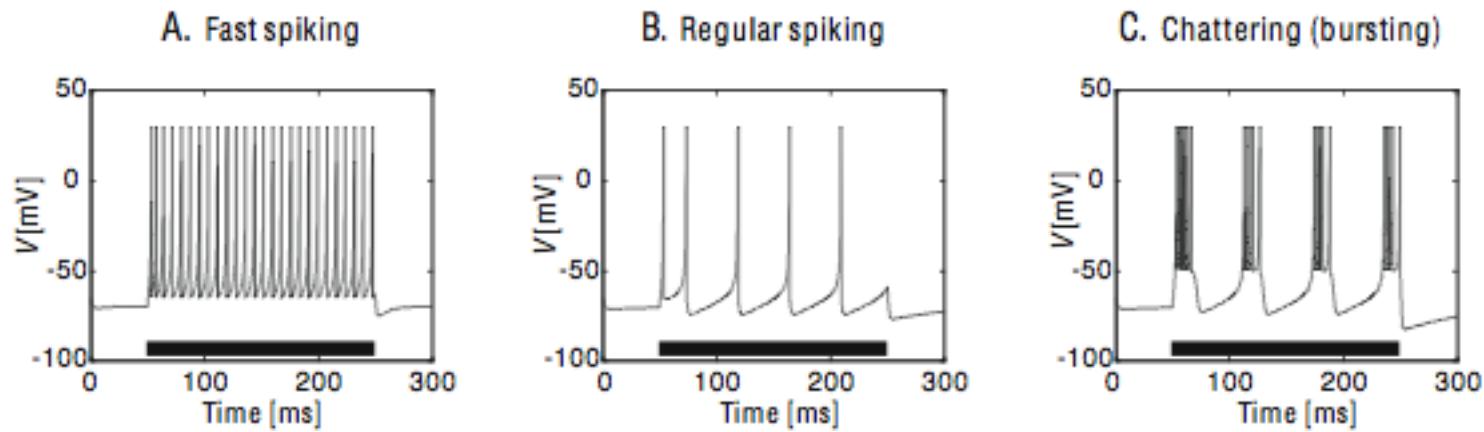


The Izhikevich neuron



$$\begin{aligned}\frac{dv(t)}{dt} &= 0.04v^2(t) + 5v(t) + 140 - u + I(t) \\ \frac{du(t)}{dt} &= a(bv - u)\end{aligned}$$

$$v(v > 30) = c \text{ and } u(v > 30) = u + d$$



The McCulloch-Pitts neuron



$$h = \sum_i x_i^{\text{in}}$$

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$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$

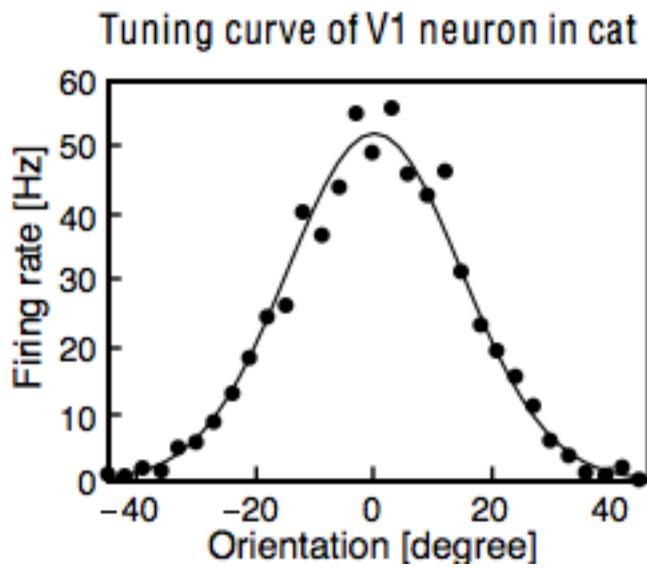
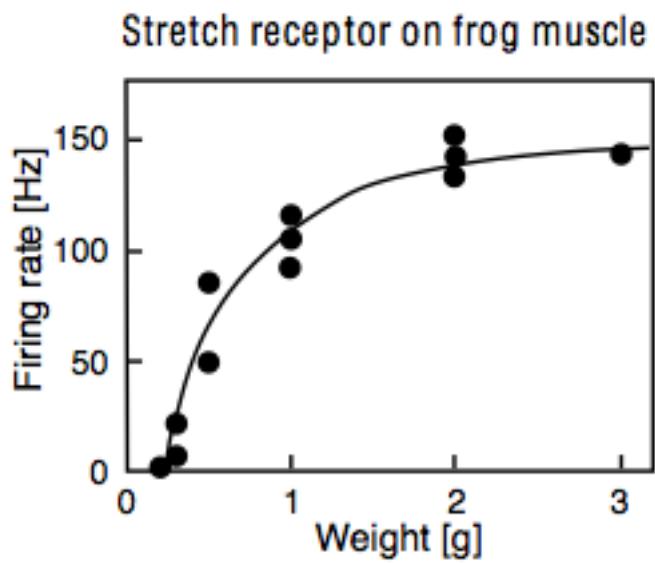
A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INSTITUTE,
AND THE UNIVERSITY OF CHICAGO

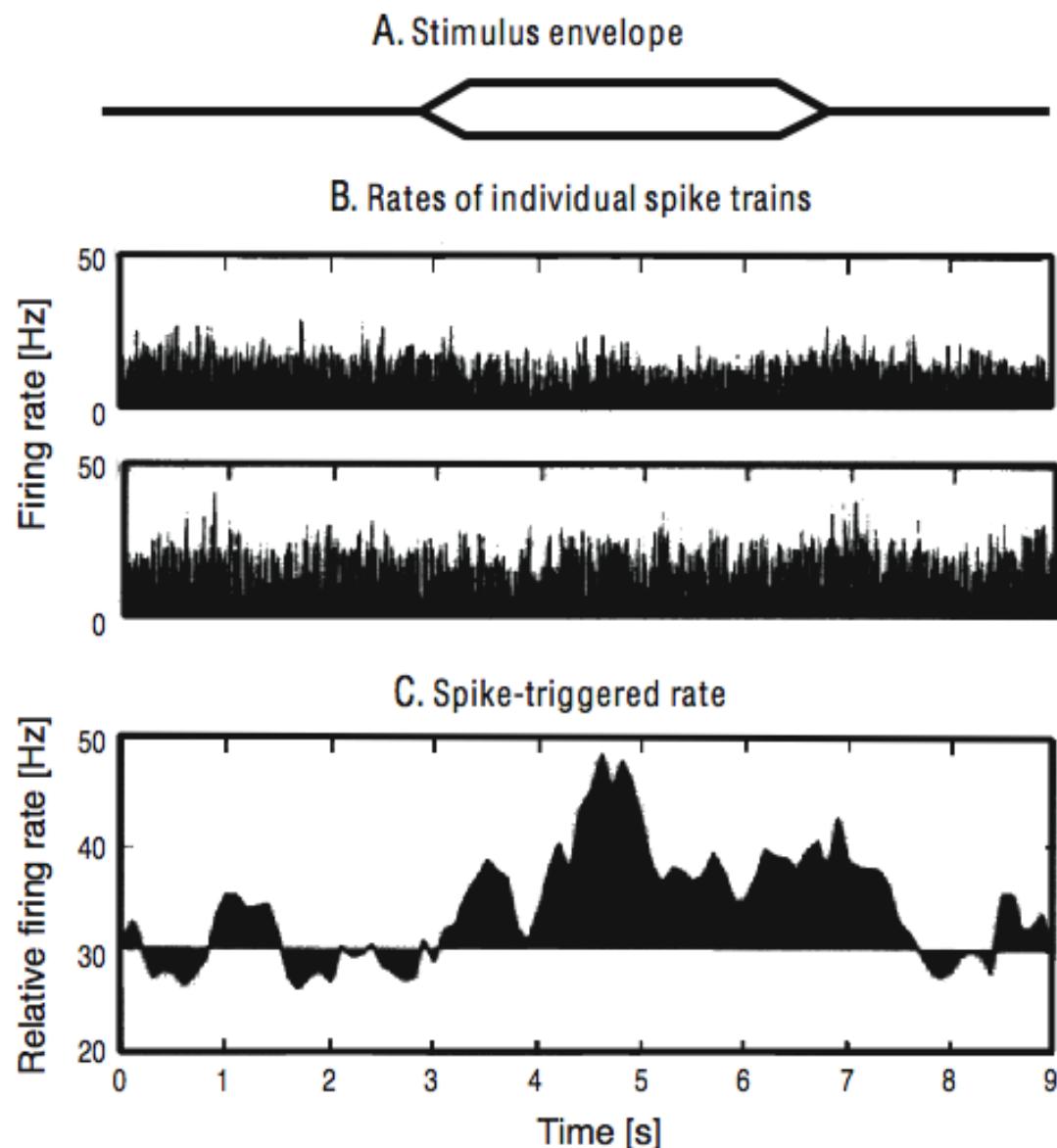
Because of the “all-or-none” character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying

The firing rate hypothesis



Edgar Adrian
The Nobel Prize in Physiology or Medicine 1932

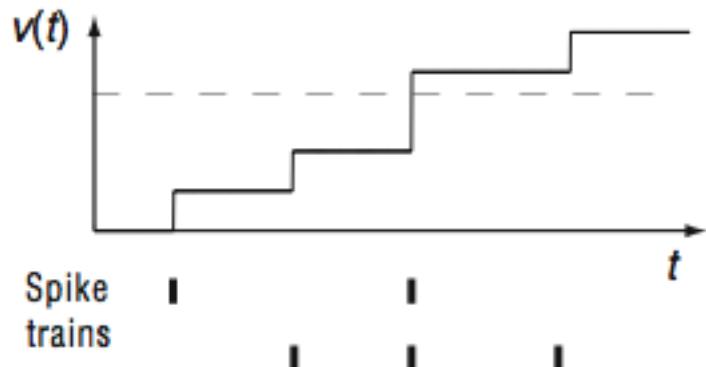
Counter example: correlation code (?)



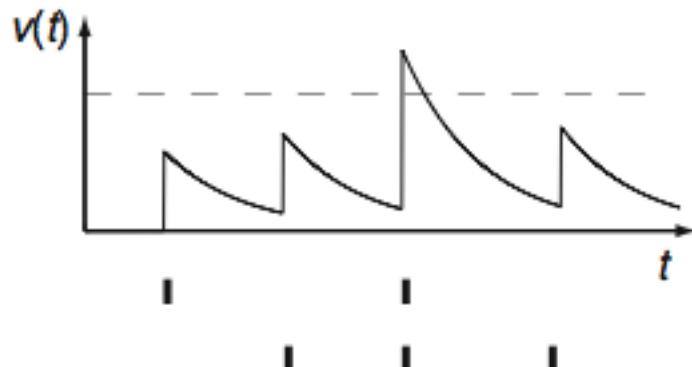
Integrator or coincidence detector?



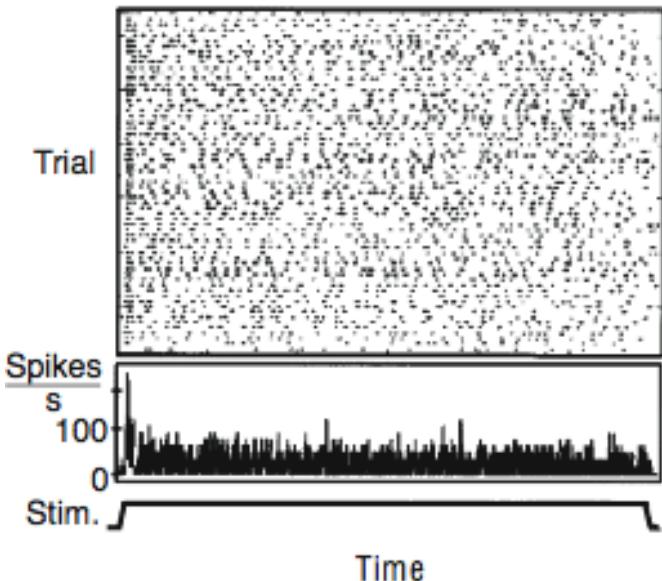
A. Perfect integrator



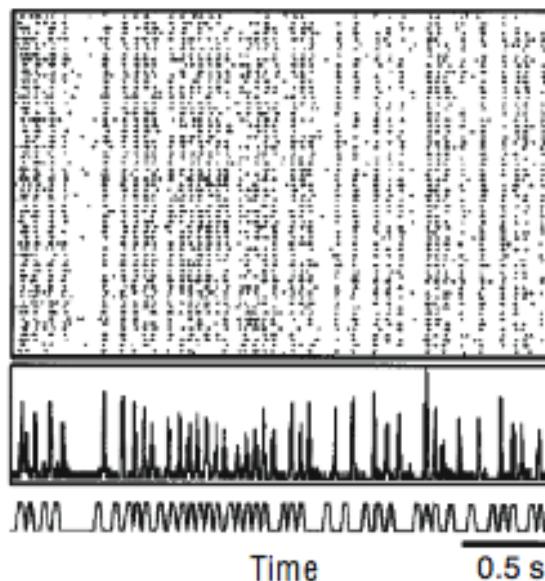
B. Coincidence detector



A. Constant stimulus



B. Rapidly changing stimulus

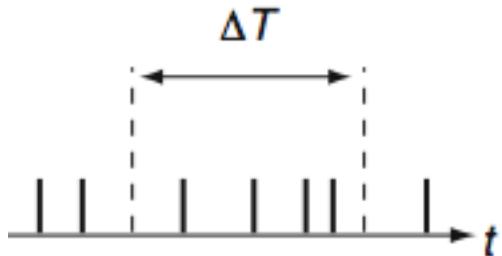


From Buracas et al. 1998

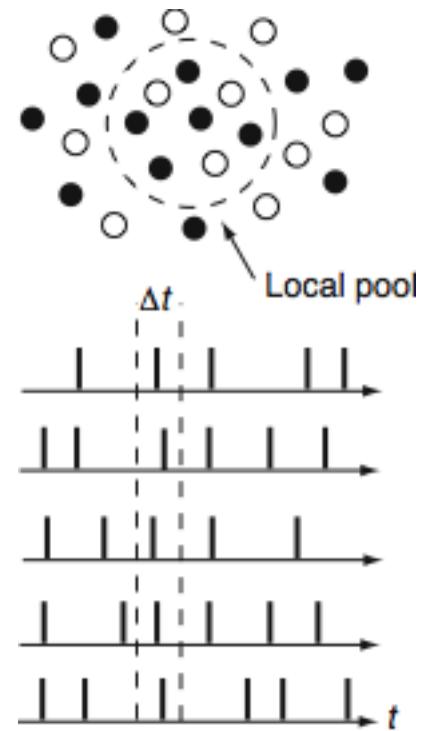
Population model



Temporal averaging



Population averaging



Population dynamics



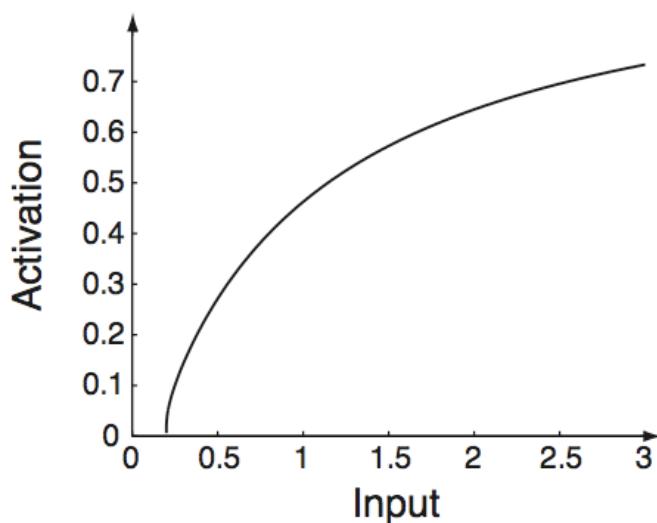
For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

$$\tau \frac{dA(t)}{dt} = -A(t) + g(RI^{\text{ext}}(t)).$$

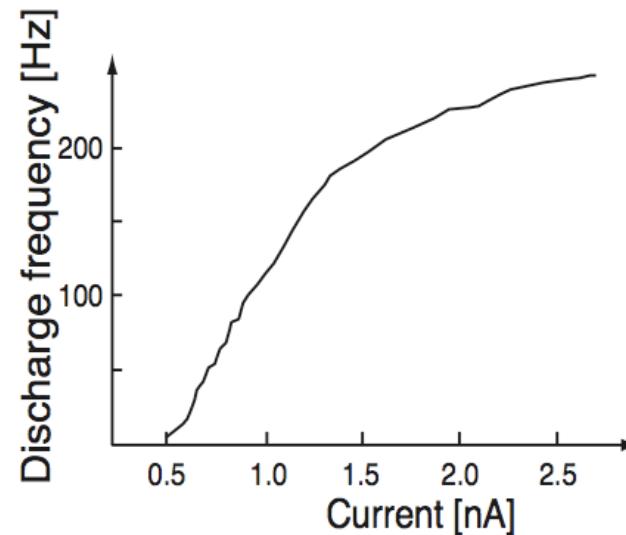
Gain function:

$$g(x) = \frac{1}{t^{\text{ref}} - \tau \log(1 - \frac{1}{\tau x})},$$

A. Activation function for population average in adiabatic limit



B. Activation function of hippocampal pyramidal neuron

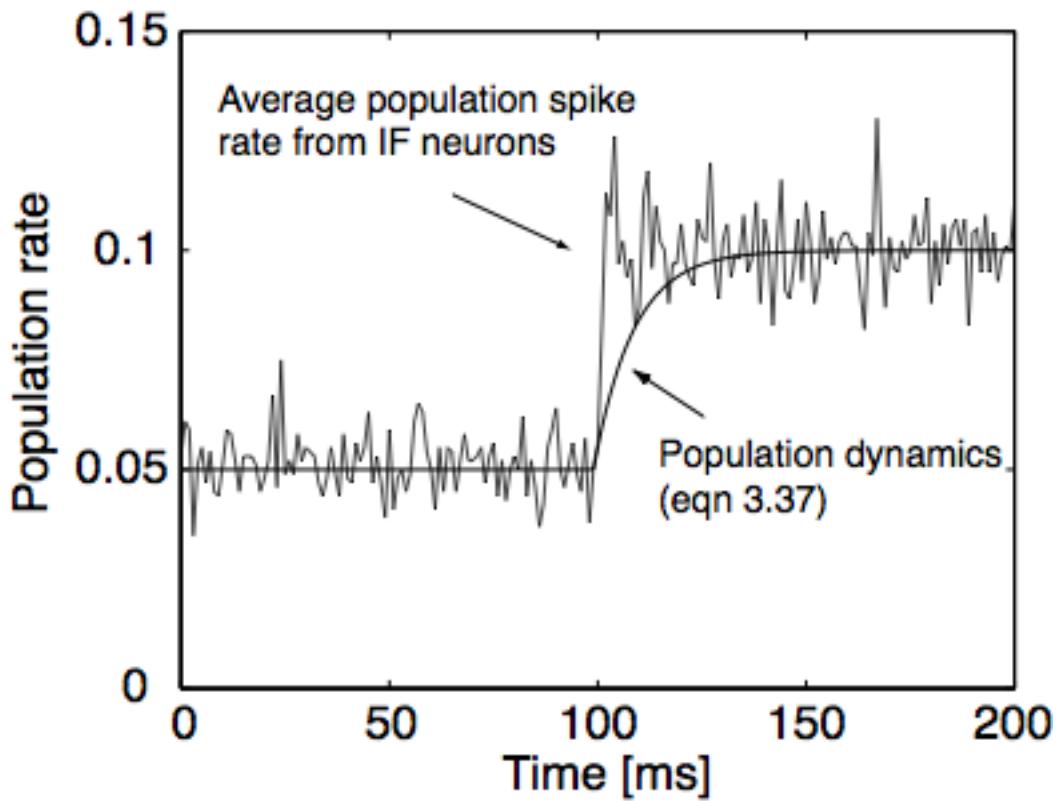


Other gain functions



Type of function	Graphical represent.	Mathematical formula	MATLAB implementation
Linear		$g^{\text{lin}}(x) = x$	X
Step		$g^{\text{step}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	<code>floor(0.5*(1+sign(x)))</code>
Threshold-linear		$g^{\text{theta}}(x) = x \Theta(x)$	<code>x.*floor(0.5*(1+sign(x)))</code>
Sigmoid		$g^{\text{sig}}(x) = \frac{1}{1+\exp(-x)}$	<code>1 ./ (1+exp(-x))</code>
Radial-basis		$g^{\text{gauss}}(x) = \exp(-x.^2)$	<code>exp(-x.^2)</code>

Fast population response



Further readings



Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.

Wolfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.

Eugene M. Izhikevich (2003), **Simple Model of Spiking Neurons**, in **IEEE Transactions on Neural Networks**, 14: 1569–1072.

Eugene M. Izhikevich (2004), **Which model to use for cortical spiking neurons?**, in **IEEE Transactions on Neural Networks**, 15: 1063–1070.

Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in **Bulletin of Mathematical Biophysics** 7:115–133.

Huge R. Wilson and Jack D. Cowan (1972), **Excitatory and inhibitory interactions in localized populations of model neurons**, in **Biophys. J.** 12:1–24.

Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.