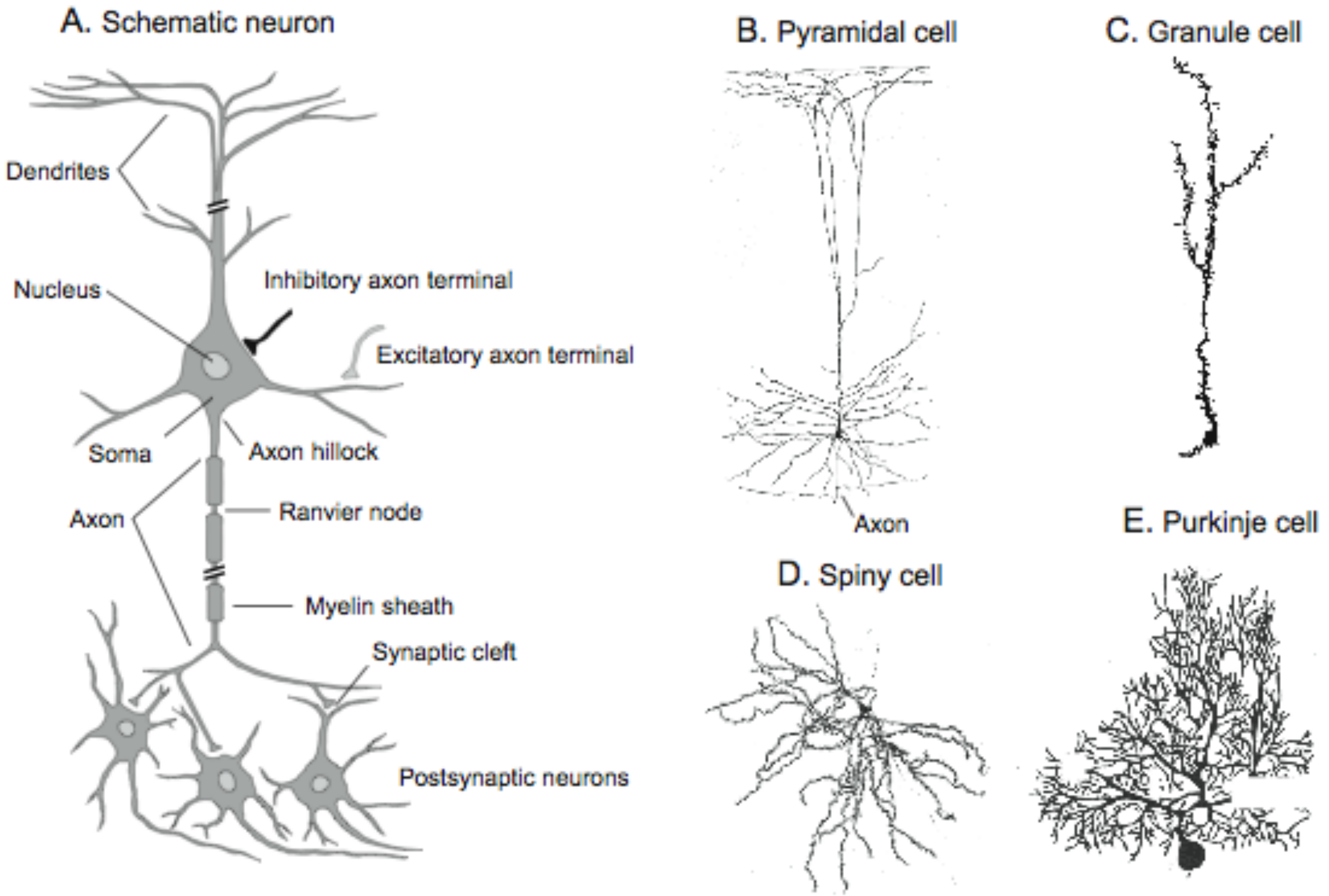
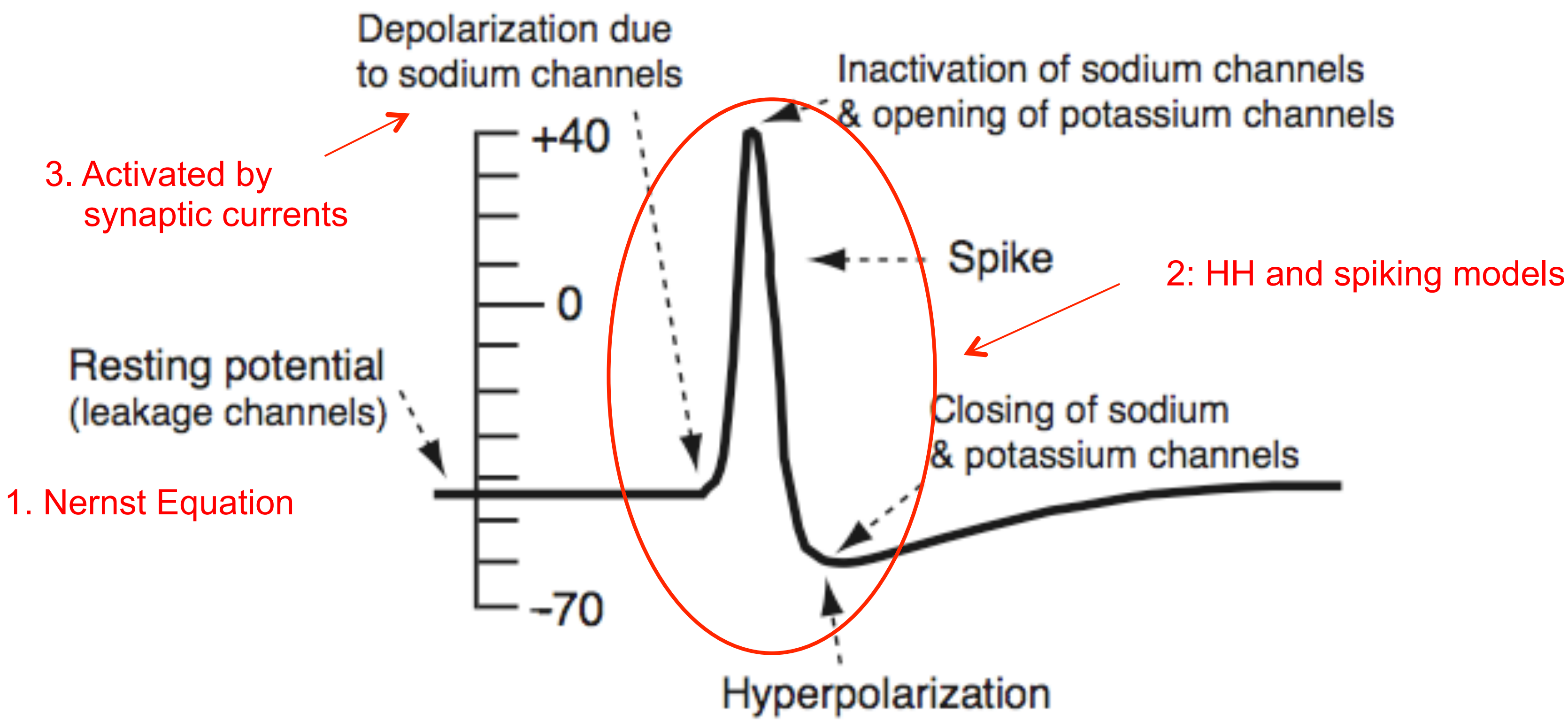
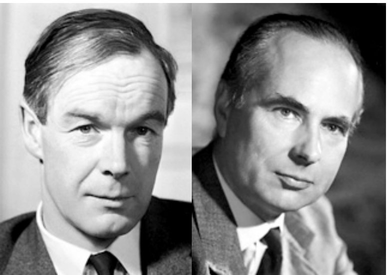
Biological background



Hodgkin-Huxley model



Typical form of an action potential; redrawn from an oscilloscope picture from Hodgkin and Huxley (1939).



Alan Hodgkin and Andrew Huxley
The Nobel Prize in Physiology or Medicine 1963 (with John Eccles)

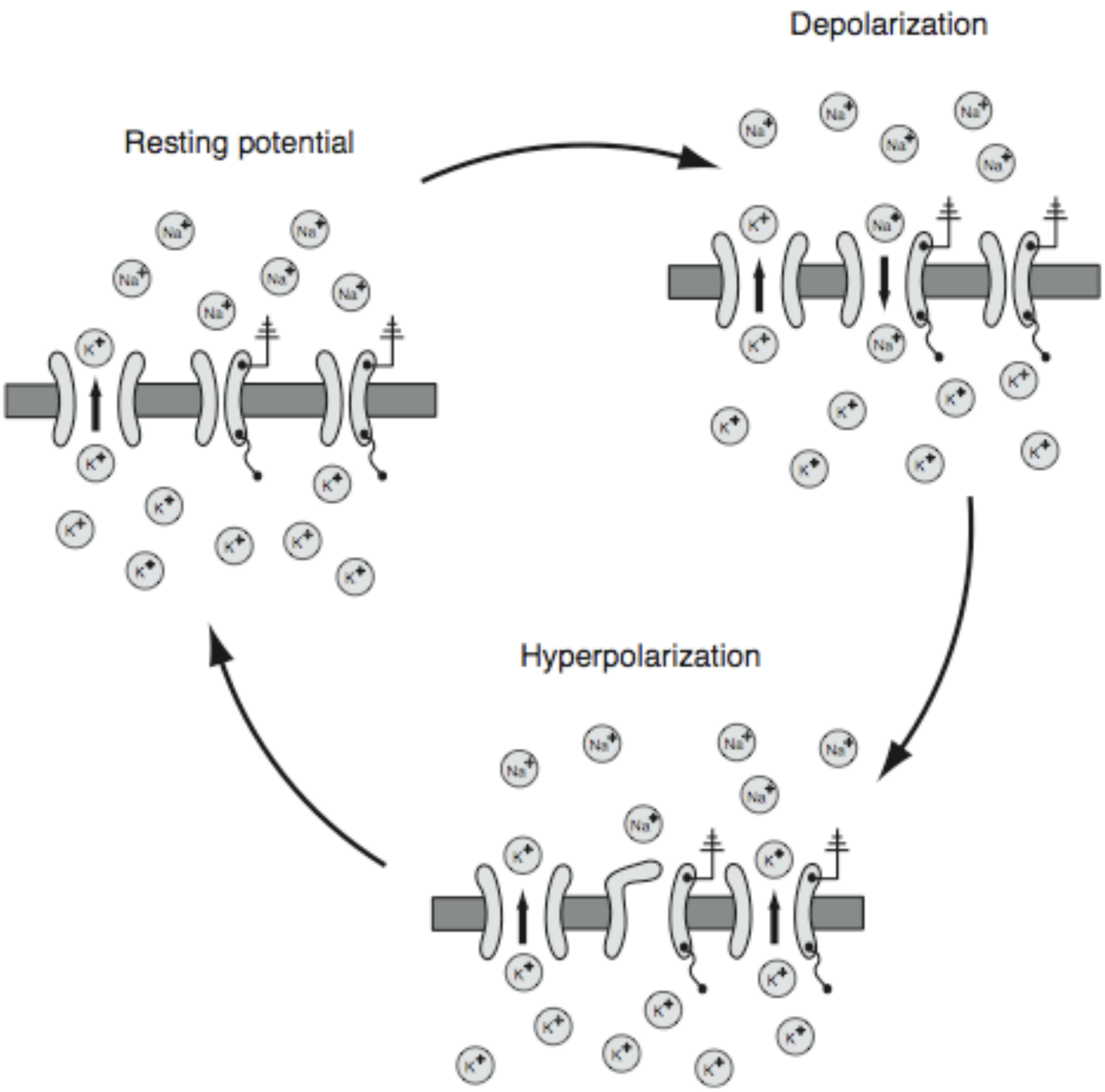
Reversal (equilibrium) potential

The reversal (or equilibrium) potential for an ion channel is the voltage (potential difference) between the outside and the inside of a cell when there is no net ion flow through this channel. This means that the electric force due to the potential difference is equal to the force due to the concentration difference of the ions inside and outside the cell. This is quantified by the

Nernst equation

$$E_{\text{ion}} = \frac{RT}{zF} \frac{[\text{ion}]_{\text{outside}}}{[\text{ion}]_{\text{inside}}}$$

The minimal mechanism



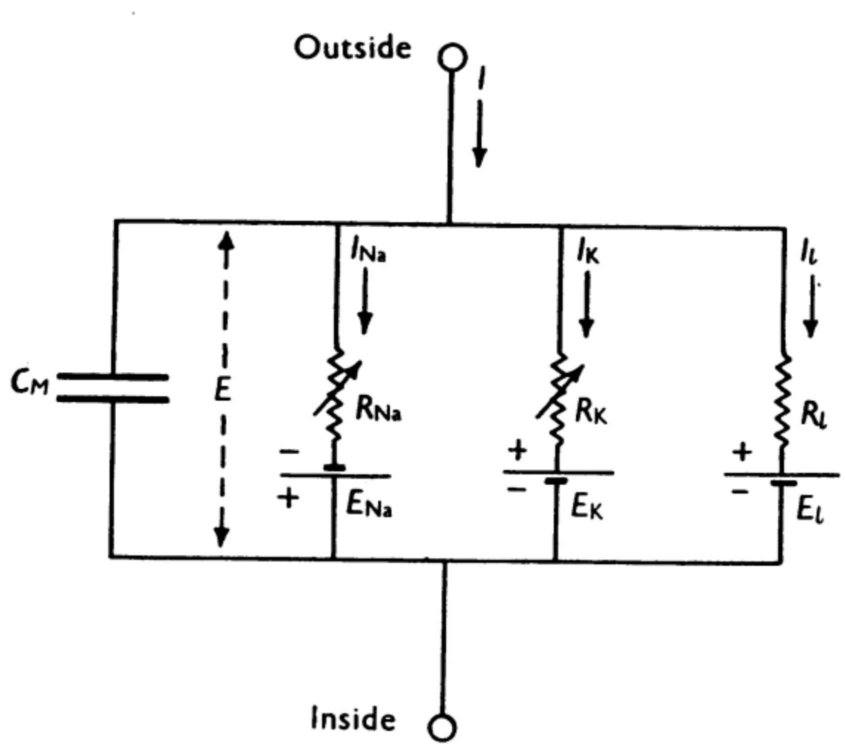
Hodgkin-Huxley equations and simulation

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = -g_{\mathrm{K}}n^4(V - E_{\mathrm{K}}) - g_{\mathrm{Na}}m^3h(V - E_{\mathrm{Na}}) - g_{\mathrm{L}}(V - E_{\mathrm{L}}) + I(t)$$

$$\mathrm{d}n/\mathrm{d}t = \alpha_n(1-n) - \beta_n n,$$

$$\mathrm{d}m/\mathrm{d}t = \alpha_m(1-m) - \beta_m m,$$

$$\mathrm{d}h/\mathrm{d}t = \alpha_h(1-h) - \beta_h h,$$



$$\alpha_n = 0{\cdot}01\,(V+10)\Big/\left(\exp\frac{V+10}{10}-1\right),$$

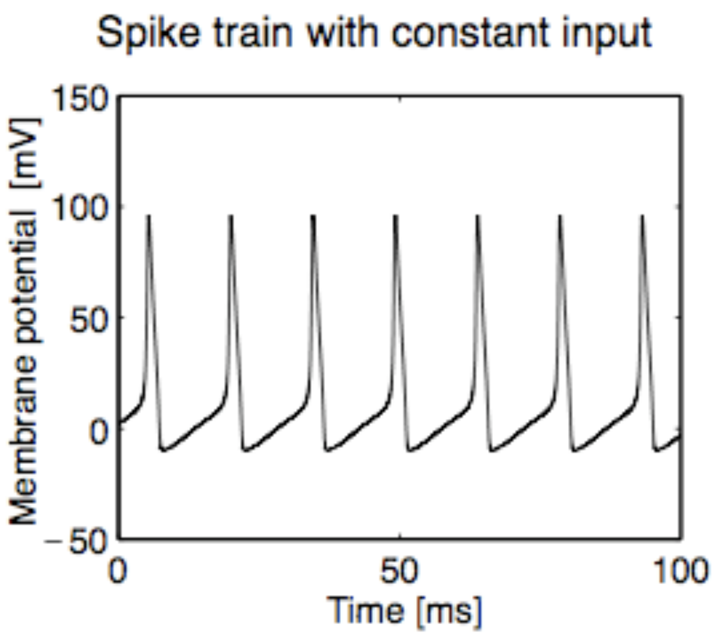
$$\beta_n = 0{\cdot}125\,\exp\,(V/80),$$

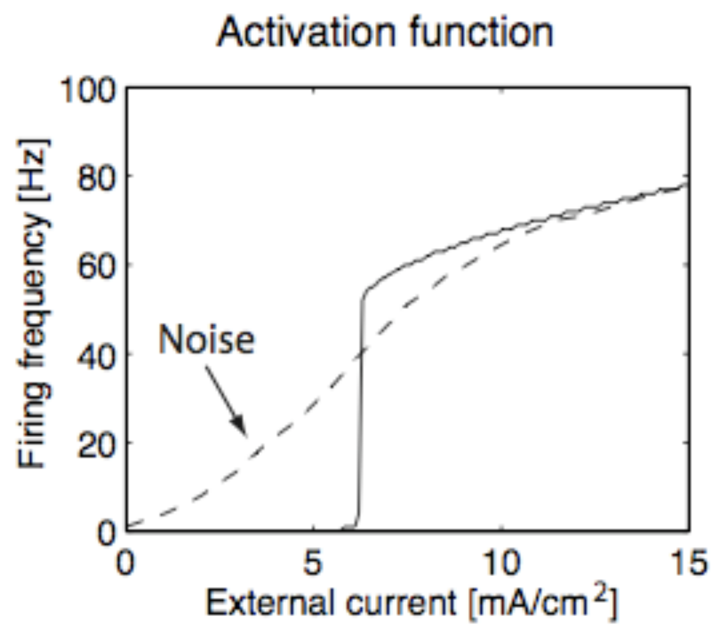
$$\alpha_m = 0{\cdot}1\,(V+25)\Big/\left(\exp\frac{V+25}{10}-1\right),$$

$$\beta_m = 4\,\exp\,(V/18),$$

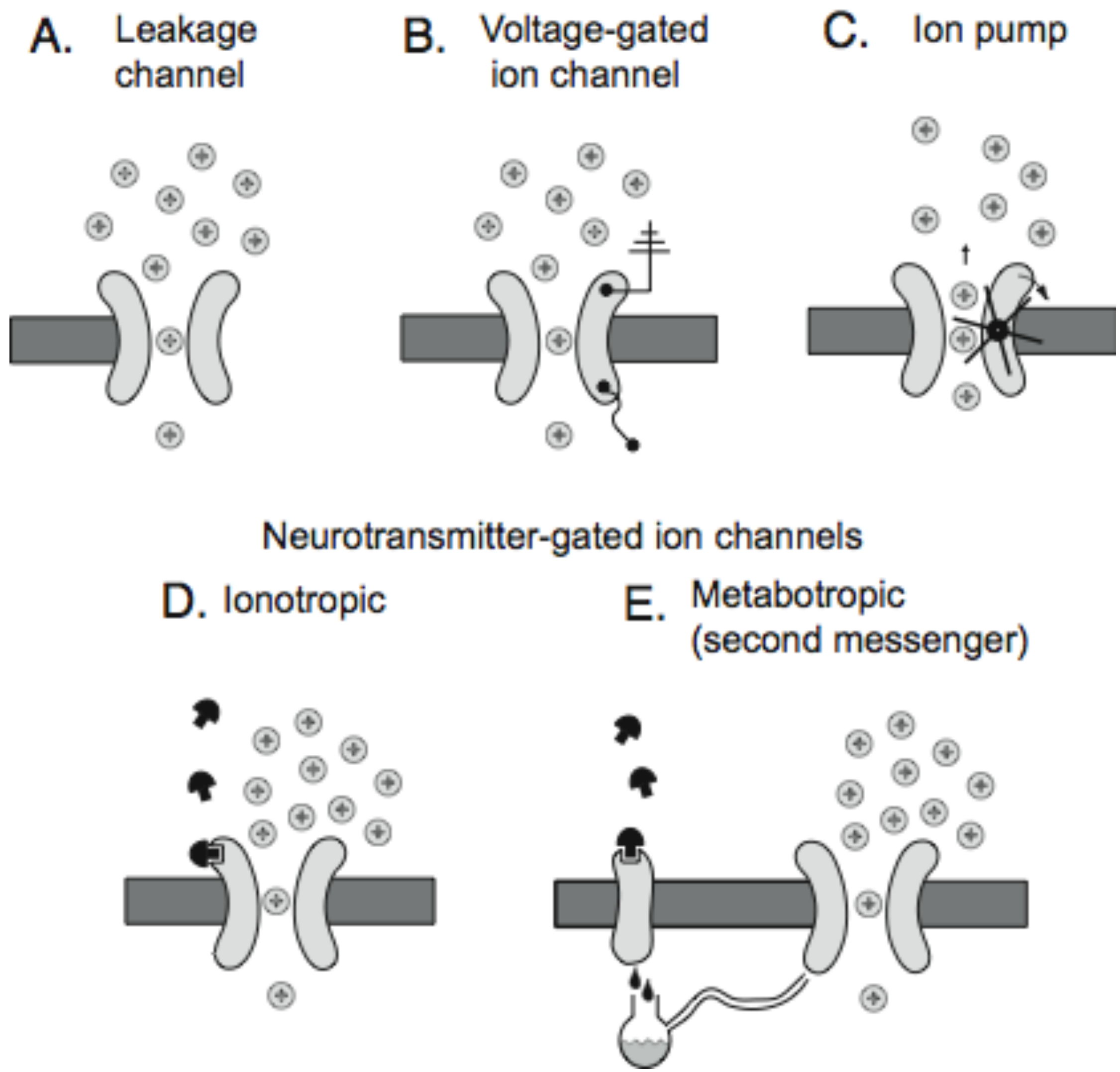
$$\alpha_h = 0{\cdot}07\,\exp\,(V/20),$$

$$\beta_h = 1\Big/\left(\exp\frac{V+30}{10}+1\right).$$

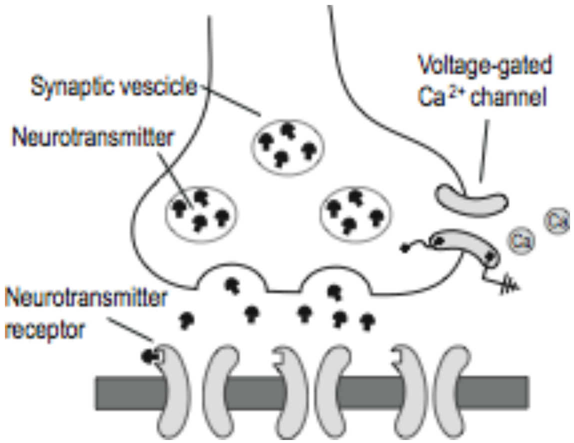




Ion channels

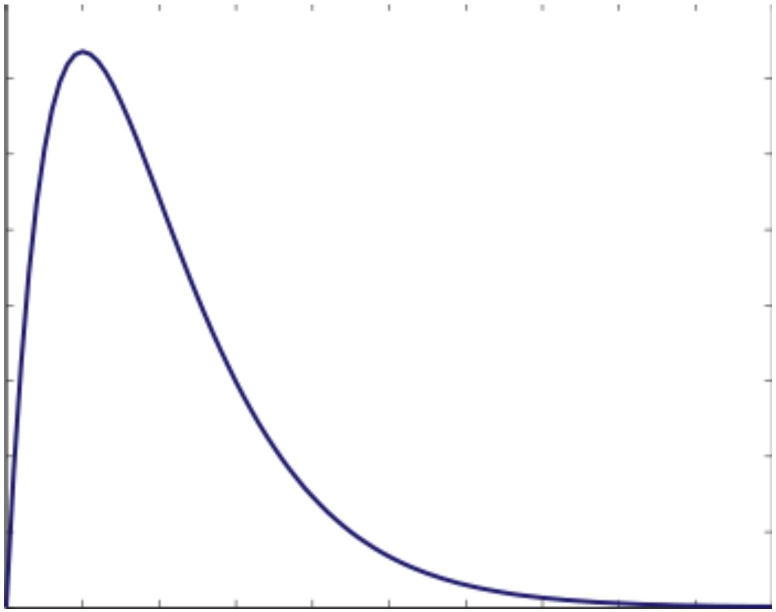


Synapse


Synaptic vescicle
Neurotransmitter
Voltage-gated
Ca 2+ channel
Ca
Neurotransmitter
receptor

Non-NMDA: AMPA, GABA

$$\Delta V_{\mathrm{m}}^{\mathrm{non-NMDA}} \propto t\, \mathrm{e}^{-t/t^{\mathrm{peak}}}$$

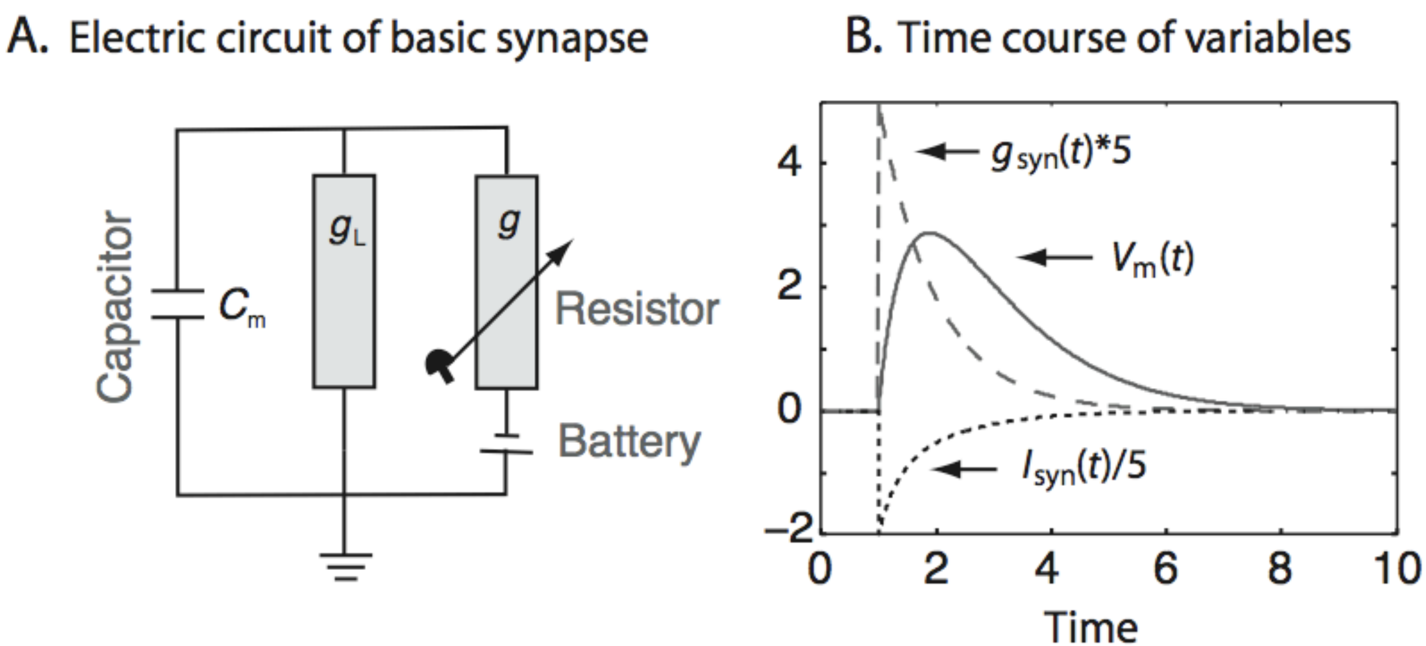


Conductance-based models

$$c_m \frac{\mathrm{d}V(t)}{dt} = -I$$

$$I(t) = g_0 V(t) - g(t)(V(t) - E_{\mathrm{syn}})$$

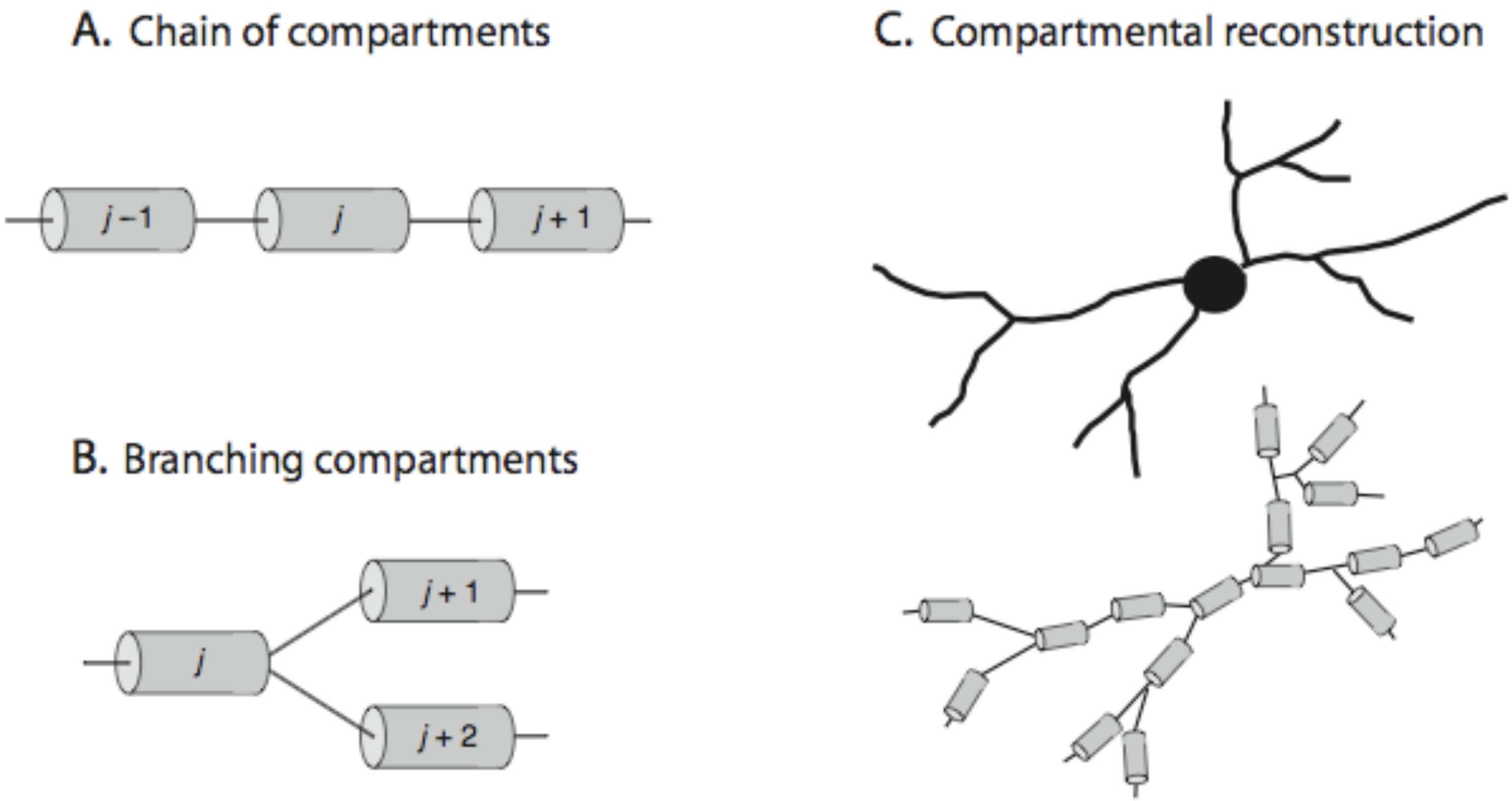
$$\tau_{\mathrm{syn}} \frac{\mathrm{d}g(t)}{\mathrm{d}t} = -g(t) + \delta(t - t_{\mathrm{pre}} - t_{\mathrm{delay}})$$



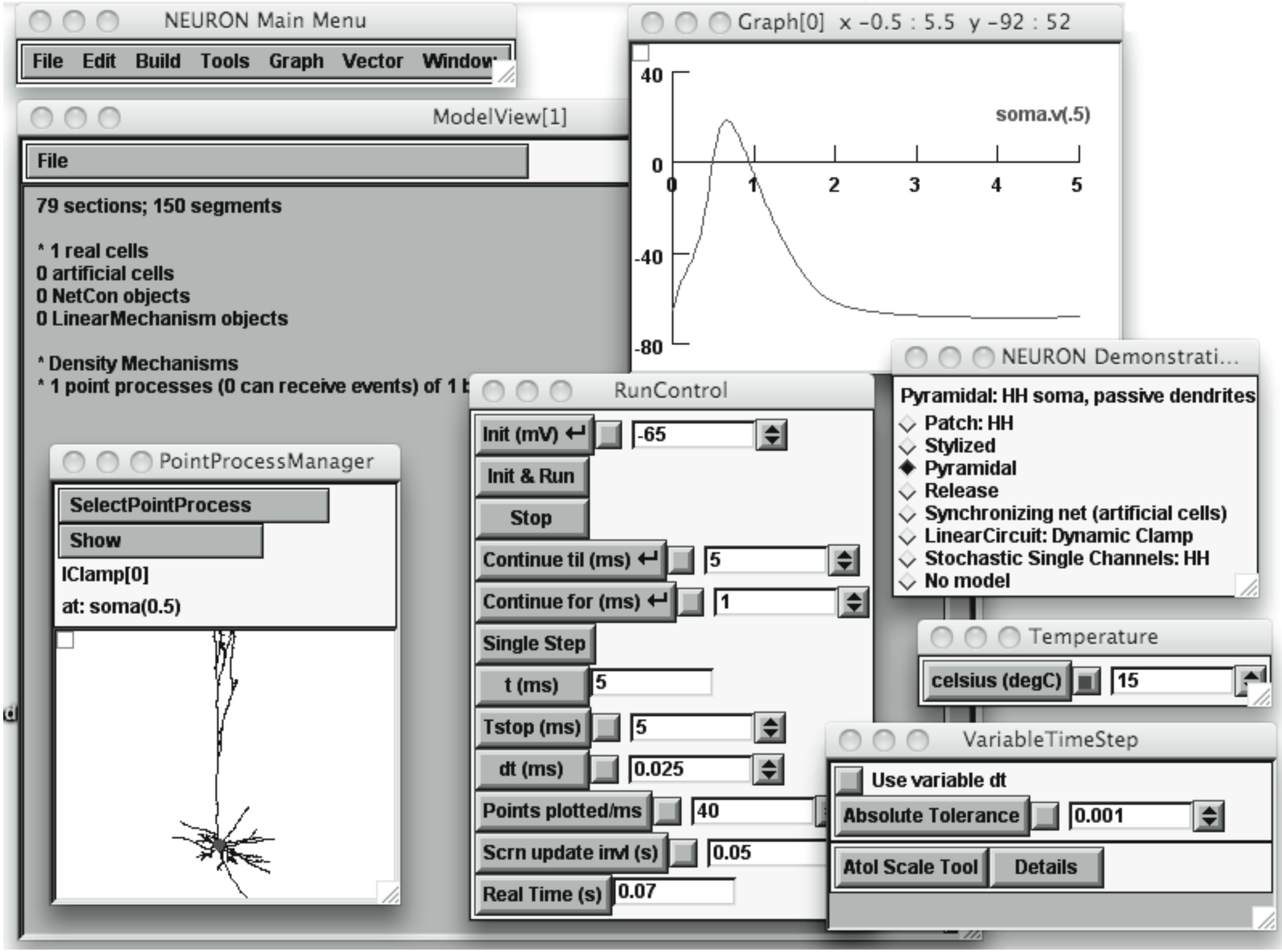
MATLAB program

```
%% Synaptic conductance model to simulate an EPSP
clear; close all; hold on;

%% Setting some constants and initial values
c_m=1; E_L=0; E_syn=10; tau_syn=1; dt=0.01;
g_L=1; g_syn(1)=0; I_syn(1)=0; v_m(1)=0; t(1)=0;

%% Numerical integration using Euler scheme
for i=2:10/dt
 % record the time (in ms) in slot i of vector t
  t(i)=t(i-1)+dt;
 % simulate the opening of the (transmitter-gated) ion chanels at t=1ms;
  if abs(t(i)-1)<0.0001;
      g_syn(i-1)=1
  end
 % calculate the currents at this time
  I_L(i)= g_L * (v_m(i-1)-E_L); % Leakage Cannel
  I_syn(i)= g_syn(i-1) * (v_m(i-1)-E_syn); % Synaptic Cannel

 % update differential equations
  g_syn(i)= g_syn(i-1) - dt/tau_syn * g_syn(i-1);
  v_m(i) = v_m(i-1) - dt/c_m*(I_L(i)+I_syn(i));
end

%% Plotting results
plot(t,v_m); plot(t,g_syn*5,'r--');
plot(t,I_syn/5,'k:');plot(t,I_L/5,'k:');
```

Compartmental models



Simulators


NEURON Main Menu
File Edit Build Tools Graph Vector Window
ModelView[1]
File
79 sections; 150 segments
* 1 real cells
0 artificial cells
0 NetCon objects
0 LinearMechanism objects
* Density Mechanisms
* 1 point processes (0 can receive events) of 1
PointProcessManager
SelectPointProcess
Show
IClamp[0]
at: soma(0.5)
Graph[0] x -0.5 : 5.5 y -92 : 52
soma.v(.5)
40
0
-40
-80
0 1 2 3 4 5
RunControl
Init (mV)
-65
Init & Run
Stop
Continue til (ms)
5
Continue for (ms)
1
Single Step
t (ms)
5
Tstop (ms)
5
dt (ms)
0.025
Points plotted/ms
40
Scrn update invl (s)
0.05
Real Time (s)
0.07
NEURON Demonstrati...
Pyramidal: HH soma, passive dendrites
Patch: HH
Stylized
Pyramidal
Release
Synchronizing net (artificial cells)
LinearCircuit: Dynamic Clamp
Stochastic Single Channels: HH
No model
Temperature
celsius (degC)
15
VariableTimeStep
Use variable dt
Absolute Tolerance
0.001
Atol Scale Tool
Details

Further readings

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Christof Koch (1999), **Biophysics of computation; information processing in single neurons**, Oxford University Press

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C. T. Tuckwell (1988), **Introduction to theoretical neurobiology**, Cambridge University Press.

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