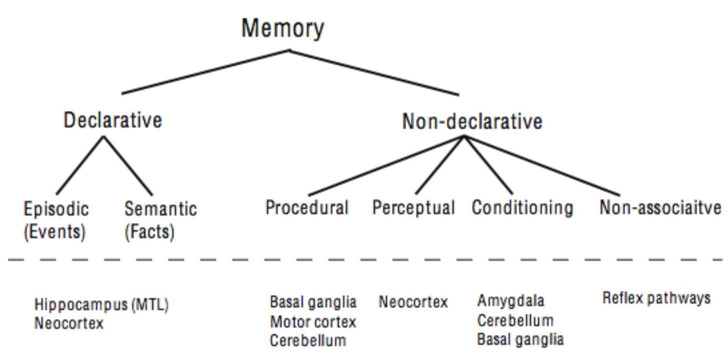
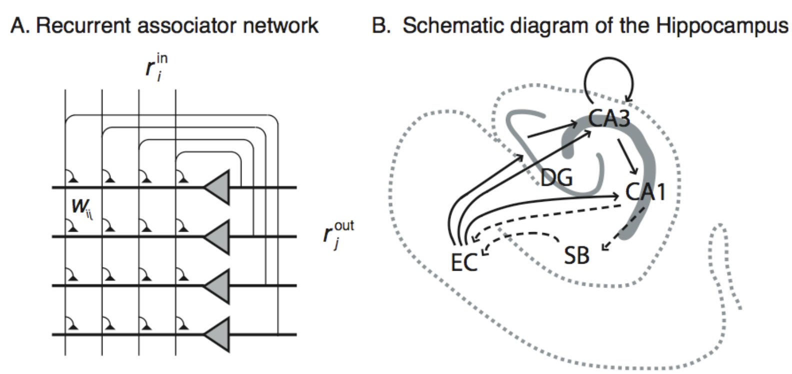
1

Memory classification scheme (Squire)



2

Auto-associative memory and the hippocampus



David Marr:
Simple memory: a theory for archicortex, 1971

3

Point attractor neural network (ANN)

Update rule: $\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + I_i^{\mathrm{ext}}(t)$

Activation function $r_i = g(u_i)$ (e.g. threshold functions)

Learning rule $w_{ij} = \epsilon \sum_{\mu=1}^{N_p} (r_i^\mu - \langle r_i \rangle)(r_j^\mu - \langle r_j \rangle) - c_i$

Training patterns: Random binary states with components
$s_i^\mu \in \{-1, 1\}$, $r_i = \frac{1}{2}(s_i + 1)$

Update equations for fixed-point model($\mathrm{d}u_i/\mathrm{d}t = 0$)**:**
$$s_i(t+1) = \mathrm{sign}\left(\sum_j w_{ij} s_j(t)\right)$$

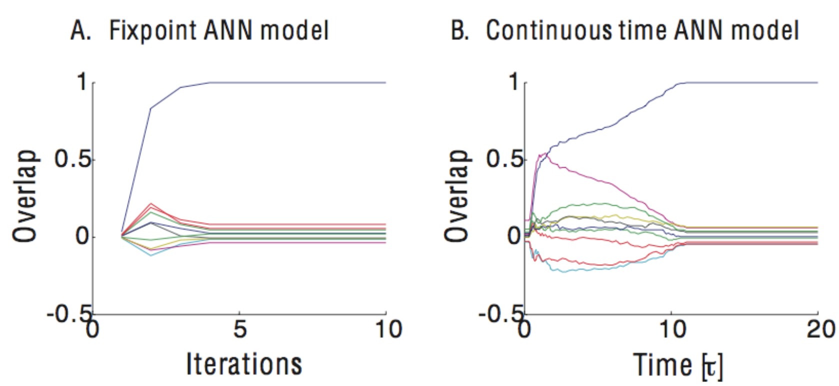
ann_cont.m

```
%% Continuous time ANN
 clear; clf; hold on;
 nn = 500; dx=1/nn; C=0;

%% Training weight matrix
 pat=floor(2*rand(nn,10))-0.5;
 w=pat*pat'; w=w/w(1,1); w=100*(w-C);
%% Update with localised input
 tall = []; rall = [];
 I_ext=pat(:,1)+0.5; I_ext(1:10)=1-I_ext(1:10);
 [t,u]=ode45('rnn_ode_u',[0 10],zeros(1,nn),[],nn,dx,w,I_ext);
 r=u>0.; tall=[tall;t]; rall=[rall;r];
%% Update without input
 I_ext=zeros(nn,1);
 [t,u]=ode45('rnn_ode_u',[10 20],u(size(u,1),:),[],nn,dx,w,I_ext)
 r=u>0.; tall=[tall;t]; rall=[rall;r];
%% Plotting results
 plot(tall,4*(rall-0.5)*pat/nn)
```


ann_fixpoint.m

```
pat=2*floor(2*rand(500,10))-1;                 % Random binary pattern
w=pat*pat';                                    % Hebbian learning
s=rand(500,1)-0.5;                             % Initialize network
for t=2:10; s(:,t)=sign(w*s(:,t-1)); end       % Update network
plot(s'*pat/500)
```



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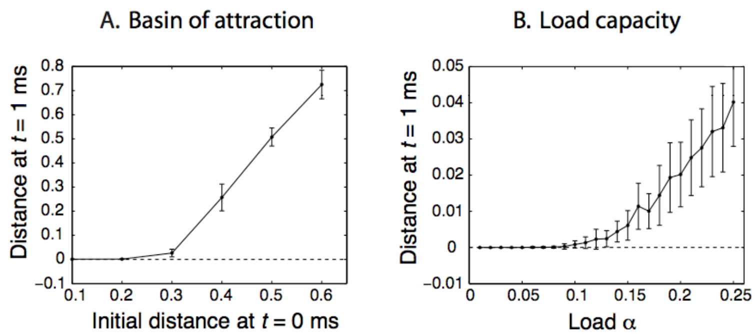
Memory breakdown



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Probabilistic RNN

Probabilistic update rule:

$$P(s_i(t) = +1) = \frac{1}{1 + \exp(-2\sum_j w_{ij} s_j(t-1)/T)}$$

Recovers deterministic rule in $\lim_{T \to 0}$

$$s_i(t) = \text{sign}(\sum_j w_{ij} s_j(t-1))$$

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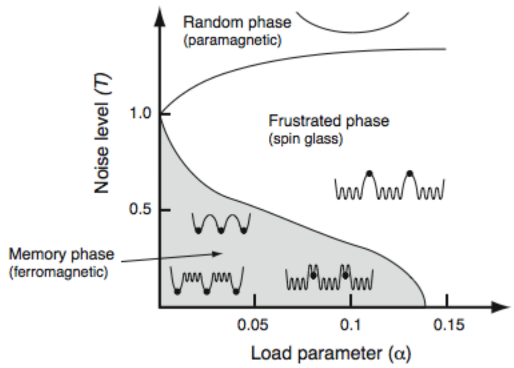
Phase diagram



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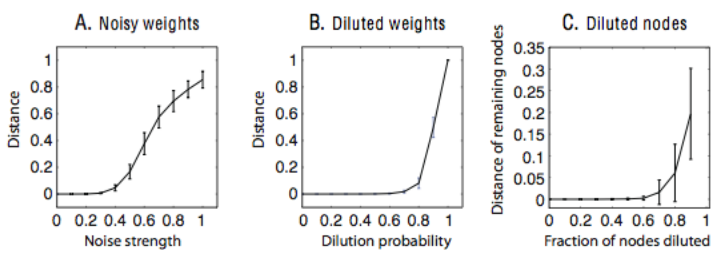
Noisy weights and diluted attractor networks



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How o minimize interference between patterns?

Associative memory in ANN is strongly influenced by interference between patter due to

- correlated patterns
- random overlap

Storage capacity can be much enhanced through decorrelating patterns. Simplest approach is generating sparse representations with expansion re-coding.

Storage capacity: $P_c \approx \frac{k}{a \ln(1/a)}$ (Rolls & Treves)

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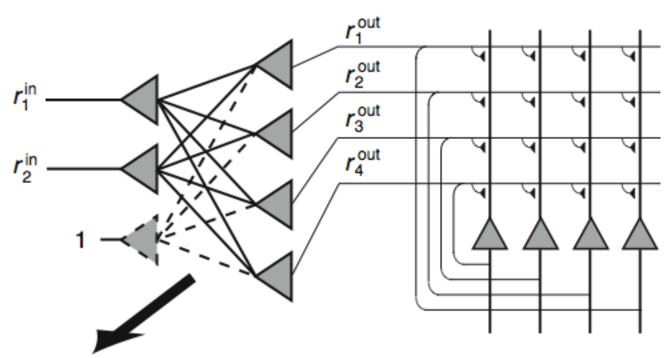
Expansion re-coding (e.g. dentate gyrus)



$$\begin{pmatrix} -1 & -1 & 0.5 \\ 1 & -1 & -0.5 \\ -1 & 1 & -0.5 \\ 1 & 1 & -1.5 \end{pmatrix}$$

r_1^{in}	r_2^{in}	r_1^{out}	r_2^{out}	r_3^{out}	r_4^{out}
0	0	1	0	0	0
1	0	0	1	0	0
0	1	0	0	1	0
1	1	0	0	0	1

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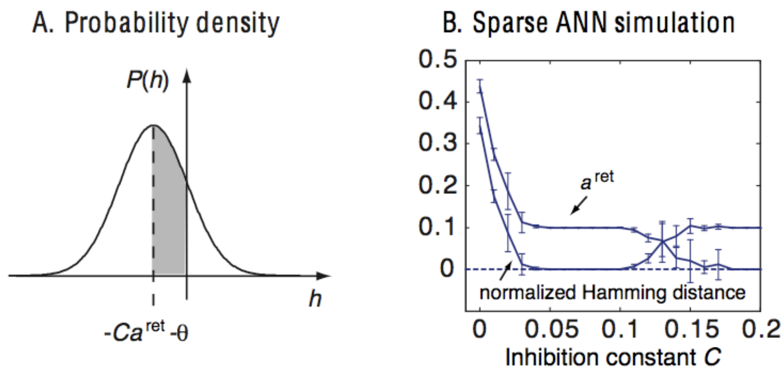
Sparse patterns



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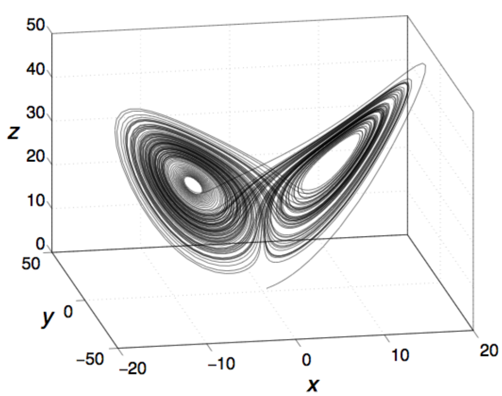
Three nodes with Σ and Σ- Π coupling: Lorenz attractor

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \sum_j w^1_{ij} x_i + \sum_{jk} w^2_{ijk} x_j x_k$$

$$\mathbf{w}^1 = \begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & c \end{pmatrix}$$

$$\mathbf{w}^2 = \begin{cases} w^2_{213} = 1 \\ w^2_{312} = 1 \\ 0 \quad \text{otherwise} \end{cases}$$



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Cohen-Grossberg theorem

Dynamical system of the form $\frac{dx_i}{dt} = -a_i(x_i)\left(b_i(x_i) - \sum_{j=1}^{N}(w_{ij}g_j(x_j))\right)$
Has a Lyapunov (Energy) function, which guaranties point attractors, under the conditions that

1. **Positivity** $a_i \geq 0$**:** The dynamics must be a leaky integrator rather than an amplifying integrator.
2. **Symmetry** $w_{ij} = w_{ji}$**:** The influence of one node on another has to be the same as the reverse influence.
3. **Monotonicity** $\mathrm{sign}(\mathrm{d}g(x)=\mathrm{d}x) = \mathrm{const}$**:** The activation function has to be a monotonic function.

→ **more general dynamics possible with:**

- Non-symmetric weight matrix
- Non-monotone activation functions (tuning curves)
- Networks with hidden nodes

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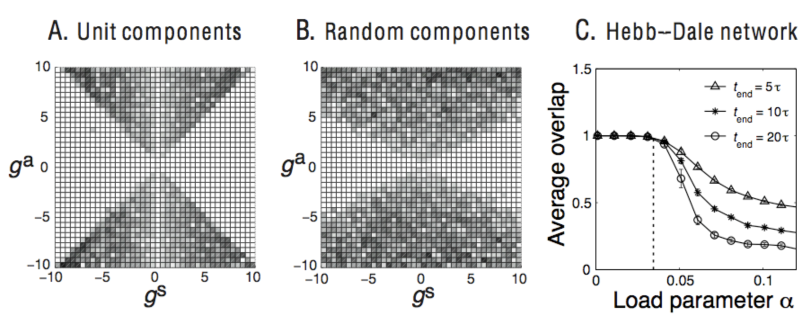
Recurrent networks with non-symmetric weights



→ **strong asymmetry is necessary to abolish point attractors**

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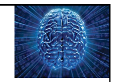

Further readings

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