

位相構造とニューラルネットワークの学習能力の相関性に関する研究

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あらまし 本研究の目的は学習システムの内部的な位相構造と学習能力の関係性を調べることである。この研究は成体の哺乳類の脳に見られる位相構造マップの学習能力と汎用性における重要性を示した神経生理学的な実験に動機付けられたものである。外部情報の組織化と学習の関連性を検証するために、自己組織化マップを中間層にもつ階層型ニューラルネットワークを用いる計算機実験を通し、情報の組織化の学習と再学習における重要性を確認した。

キーワード 自己組織化マップ (SOM), トポロジー表現, 皮質可塑性, 教師なし学習, 教師あり学習

Relation between Topological Organization and Learning Ability of Neural Networks

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Abstract The objective of this study is to investigate the correlation between the internal topological organization in neural network and the learning ability of the neural network. This study is motivated by the interesting neurophysiological examination that shows the significance of topographic map of adult mammals' brains to their learning ability and plasticity. In this study we propose a model of a layered neural network with Self-Organizing Map in its hidden layer which is connected to Perceptron as a learning part. We run several simulations to show the significant of the topological order in helping the learning process and relearning process.

Key words Self-Organizing map (SOM), Perceptron, Topological Representation, Cortical Plasticity, Unsupervised Learning, Supervised Learning

1. Introduction

Self-Organizing map has been a principle model for explaining experience-driven development and leaning in the brain ([1], [2], [3], [4], [5], and [6]). This paper provides initial experiments on the relation between the topological representations the external stimuli and the learning ability of a learning system.

This work is partly motivated by the existence of a biological experiment in investigating the significance of the topographic map of adult mammals' brain for learning ([7], [8], and [9]).

For investigating the correlation between topological order and the learning ability of a leaning system, we modified a layered neural network model called Map Initialized Perceptron (MIP) ([11] and [12]). This layered model includes a topological map in its hidden layer to internally represent and organize the external stimuli, and a feed forward layer (Perceptron layer). We are aware that there are several models that inherently combined self-organization and learning as in [13] and [14]. However, learning in these models is inherently included in the self-organization process, hence it will be difficult to utilize this model for our study, which requires the investigation on the relation between the fidelity

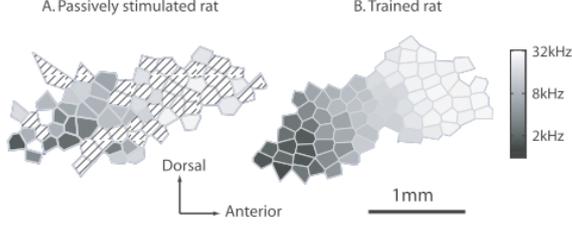


图 1 Sound frequency maps in rat A1 that was developed in noisy environments. (A) shows a map after the rat grew up. The diagonally hatched areas were not tuned. (B) shows a map after the rat was trained. The rat did tasks to distinguish sound frequency to get food. ([10])

of topological organization with the learning ability.

This paper is organized as follows. Section 2 explains about the dynamics and learning process of MIP. In Section 3 we run experiments to investigate the significance of topological organization to the learning ability of the model. We also argued about the importance of the topological fidelity by gradually distant the representation of the topological layer through the deletion of its hidden neurons. Here, plasticity and the relearning ability of the model is also explained. Final Section is for discussion and future works.

2. Map-Intialized Perceptron

For the purpose of investigating the correlation between the fidelity of topological representation and the learning ability of neural networks, we utilize a three-layered neural network model called Map initialized Perceptron [10], [11]. Similar to MLP [15], MIP consists of input, hidden and output layers. The input layer receives external sensory inputs, which are then topological mapped into the hidden layer. The neurons in the hidden layer generate outputs which reflects the topological characteristics inherent in that layer and forward the information to be processed by the output layer.

The structure of MIP is illustrated in Fig. 2, where the hidden layer is one dimensional topographic map.

The training process of MIP consists of two stages, unsupervised training for the formation of topographical map in the hidden layer and the supervised training for generating external outputs. There is no limitation on the training problems, but for clarity and simplicity, in this report we provide the model with a basic task of coding analog signals observed in the input layer, into their digital representations in the output layer.

2.1 Dynamics of MIP

Here the task of MIP is to decode continuous value between 0 and 2π given as input into their digital representation in the output layer. For this task we set the number of hidden

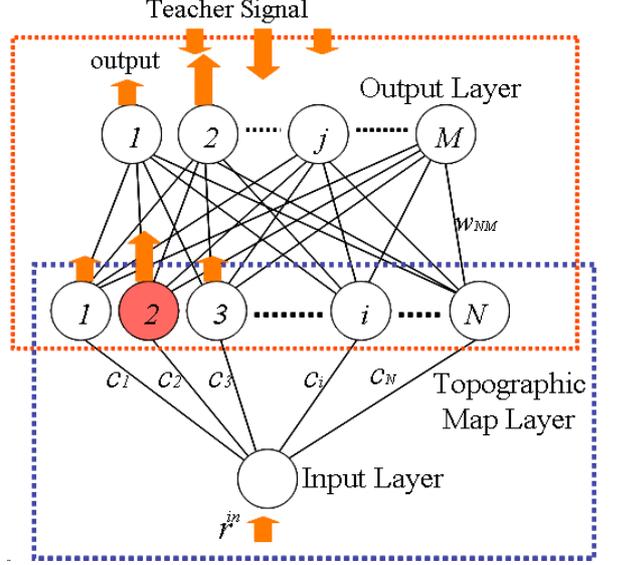


图 2 Overview of MIP

neurons to 150 and the output neurons to 100, respectively. The hidden neurons are evenly intervals in one dimensional grid to form a one dimensional topographic map. To avoid distortions in topological representations of the edges of the input signal, we use a cyclic (periodic) map in the hidden layer, where the distance between the first hidden neuron and the last one is assumed to be 1.

When MIP observes an input $r^{in}(t)$, it measures the distance between the input and the reference vector $c_i(t)$ encoded by the i -th hidden neuron, $d_i(t)$ as follows.

The winner is chosen followed Eq. 2. R is the max value of input range.

$$d_i(t) = \min(|c_i(t) - r^{in}(t)|, R - |c_i(t) - r^{in}(t)|) \quad (1)$$

$$win = \arg \max_i (\exp(-d_i(t)^2)) \quad (2)$$

The reference c_i is then modified according to Eq. 3.

$$c_i(t+1) = c_i(t) + \alpha_{som} \Lambda(d_{win}(t)) dif_i(t) \quad (3)$$

Where dif_i is the modification value defined in Eq. 4.

$$dif_i(t) = \begin{cases} -(I - (r^{in}(t) - c_i(t))) & (r^{in}(t) > c_i(t) \wedge r^{in}(t) - c_i(t) > \frac{I}{2}) \\ +(I + (r^{in}(t) - c_i(t))) & (r^{in}(t) < c_i(t) \wedge c_i(t) - r^{in}(t) > \frac{I}{2}) \end{cases} \quad (4)$$

After the formation of organization map, the second stage of the learning process is executed. In this stage, the connections between the hidden layer and the output layers are trained in a supervised manner.

The value of the j -th output at time t , o_j is calculated according to Eq. 5. Here, R is the maximum value of the input.

$$o_j(t) = \sum_{i=1}^N (r_i^{out}(t) \cdot w_{ij}(t)) \quad (5)$$

$$r_i^{out}(t) = \exp\left(\frac{-d_i(t)^2}{2(\eta)^2}\right) \quad (6)$$

The object of the learning is to minimize the error, E , defined as follows.

$$E(t) = \sum_j (o_j^{out}(t) - T_j(t))^2 \quad (7)$$

T_j is the teacher signal for the j -th output neuron defined in Eq. 8 and Eq. 9.

$$d_j^t(t) = \min\left(\left|j \cdot \frac{R}{M} - r^{in}(t)\right|, \left|R - \left|j \cdot \frac{R}{M} - r^{in}(t)\right|\right\right) \quad (8)$$

$$T_j(t) = \exp\left(\frac{-d_j^t(t)^2}{2(\eta)^2}\right) \quad (9)$$

The modification of the connection weights is executed according to Eq. 10.

$$w_{ij}(t+1) = w_{ij}(t) + \alpha \cdot (T_j(t) - o_j(t)) \cdot r_i^{out}(t) \quad (10)$$

Here, η is the learning rate which is empirically set as 0.1.

3. Experiments

3.1 Effect of topological order to the learning ability

For analyzing the effect of the topological order to the learning ability, we run three experiments. In the first one, we executed the Perceptron learning in MIP without organizing a topological map in the hidden layer. In this case, the external input are not topological represented in the hidden layer. In the second experiment, the topographic map was to some extent trained prior to the execution of the Perceptron learning, while in the final case, the topographic map was exhaustively trained prior to the Perceptron learning.

Fig. 3 shows the learning curve of Perceptron training. We can clearly observe that the Perceptron layer can be rapidly trained when the fidelity of the topological organization is high. We can learn that topological order in the hidden layer significantly helps the training process.

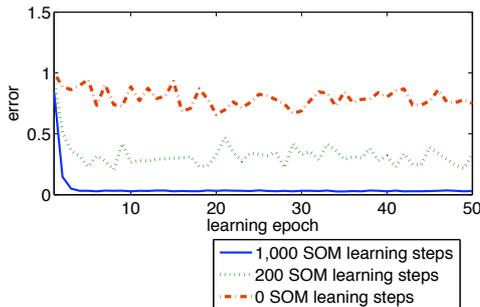


Fig. 3 Topological Order and Learning Ability

3.2 Reorganization and Retraining

In this experiment we analyzed the fidelity of MIP, when the topological order in the hidden layer is disturbed. We simulate the disturbance by randomly removing a certain number of hidden neurons. This has a coarse biological analogy with a partially damaged brain. The objective is to observe how the reorganization and relearning can help to recover the function of the learning system.

In these experiments, we removed 5-95% of hidden neurons in a consecutive manner, and executed two types of relearning. In the first case, the map in the hidden layer is reorganized to capture alternative topological order using the rest of the neurons, prior to the retraining of Perceptron. The first case is denoted as “SOM and Perceptron” in Fig. 4. In the second case, Perceptron is retrained without reorganizing the topographic map in the hidden layer.

The experiments results are given in Fig. 4, where we can observe that when the percentage of the damaged neurons exceeds 15 %, the training quality of the Perceptron gradually deteriorates if the topographic map is not reorganized. This result is understandable, in that, the topographic map does not have sufficient ability to represent the underlying topological characteristics of the external inputs. We can also observe that reorganization in the hidden layer helps to support the retraining process until the number of the damaged neurons reaches a level where the hidden layer lost its ability to topologically represent the input. ”Error of the Center of Mass” in Fig. 4 refers to the defraction between the output neurons and their teacher signals, taking the cyclic characteristics into account.

These experiments shows that topological order is crucial in supporting not only the supervised learning process but also the relearning process.

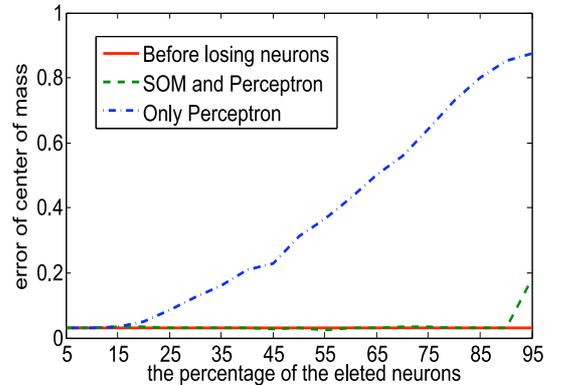


Fig. 4 Disturbance in Topological Representation

Fig. 5 shows the learning curve of Perceptron with regards to various percentages of the damaged neurons. These graphs show that the neural network can sustain its functionality although it lost 80% of the hidden neurons as long as

the topographic map in the hidden layer is reorganized using the rest of the neurons. It is also clear that relearning of Perceptron without reorganization fails when the percentage of the damaged neurons exceeds 50%.

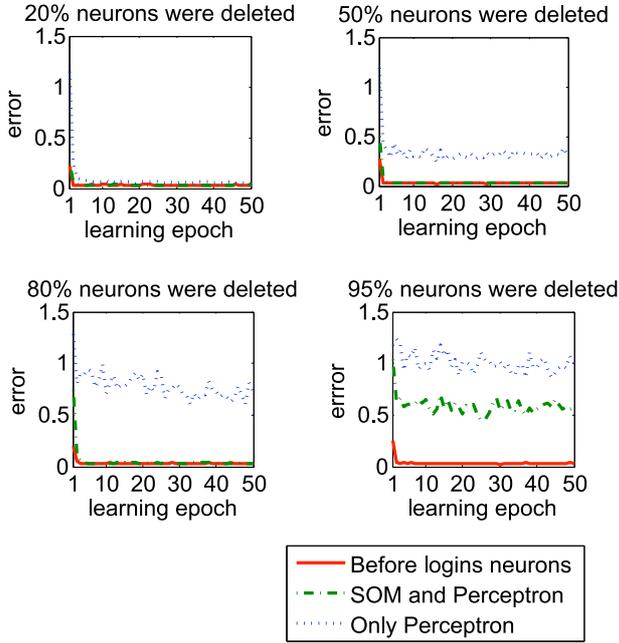


图 5 Topological Disturbance and Learning Ability

Fig. 6 shows the errors of the output neurons with respect to the teacher signals, in the case of various percentages of damaged neurons.

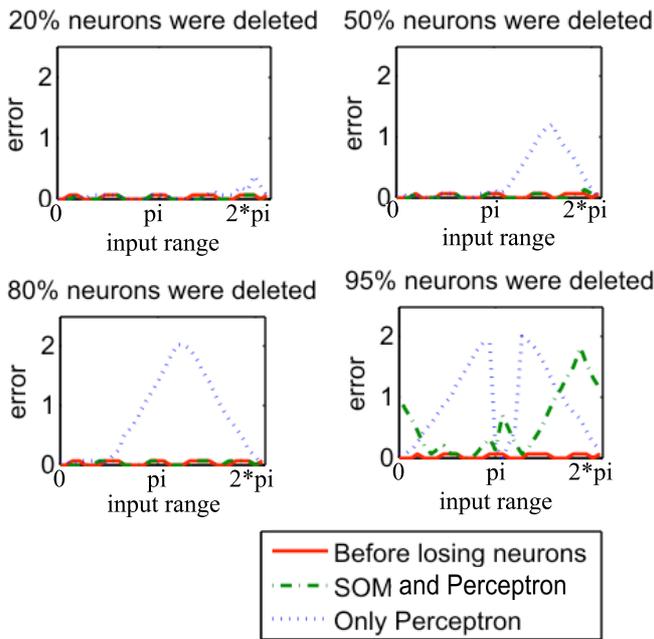


图 6 Topological Disturbance and External Output

Fig. 7 shows the reference value, c_i of the respective hidden neurons. It is obvious that after the number of damaged

neurons reached a critical threshold, the neural network does not have sufficient power to represent the input, and consequently lost its ability to function properly.

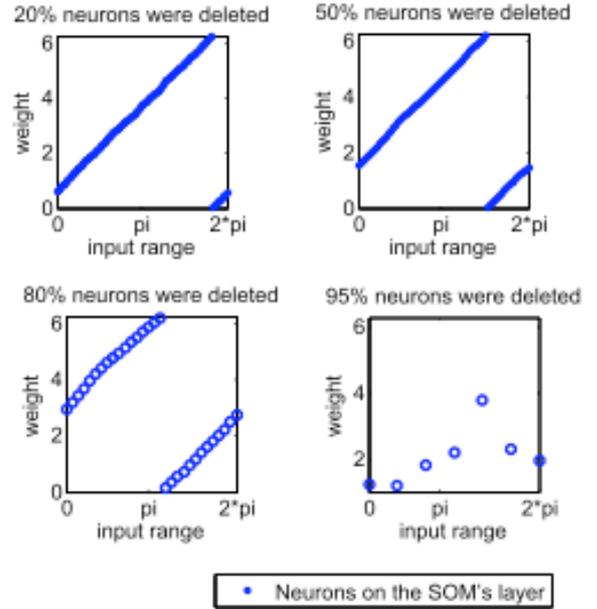


图 7 Lost and Hidden Neurons and Topological Representation

4. Conclusion and Future Work

In this study, utilizing a modified MIP we run several simulations for arguing the importance of topological representation and the learning ability of a learning system. Our experiments showed that the quality of the topological representation helps the neural network in forwarding the supervised learning process. The results also show the plasticity of the topological representation and the learning system, where relearning of a partially damaged representation is possible as long as the internal topological organization is not excessively damaged.

Our immediate future works is to implement a physiological topological map model in the middle layer of MIP. We plan to utilize Dynamic Neural Field (DNF) [2], which, unlike conventional SOM which only works with one input, can work with multiple inputs. We are interested in investigating the quality of the self-organization in the existence of top-down attention and also noise, and its relation with leaning ability.

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